UNIT 23 EXERCISES 6-10

FUNCTIONS

2003A 6. Define $x\heartsuit y$ to be $|x-y|$ for all real numbers x and y. Which of the following statements is **not** true?

(A) $x\heartsuit y = y\heartsuit x$ for all x and y

(B) $2(x\heartsuit y) = (2x)\heartsuit(2y)$ for all x and y **(C)** $x\heartsuit 0 = x$ for all x

(D) $x\heartsuit x = 0$ for all x **(E)** $x\heartsuit y > 0$ if $x \neq y$

2017A 7. Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017)$?

(A) 2017 **(B)** 2018 **(C)** 4034 **(D)** 4035 **(E)** 4036

2003B 8. Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1+2+3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 10

2001

9. Let f be a function satisfying $f(xy) = f(x)/y$ for all positive real numbers x and y. If $f(500) = 3$, what is the value of $f(600)$?

(A) 1 **(B)** 2 **(C)** $\frac{5}{2}$ **(D)** 3 **(E)** $\frac{18}{5}$

2003B

9. Let f be a linear function for which $f(6) - f(2) = 12$. What is $f(12) - f(2)$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36