UNIT 18 EXERCISES 6-10

WORD ALGEBRA

1999

6. What is the sum of the digits of the decimal form of the product $2^{1999} \cdot 5^{2001}$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 10

2001

6. A telephone number has the form $ABC-DEF-GHIJ$, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2002A

6. For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \leq m + n$?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** infinitely many

2008B 6. Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?

(A) 2500 **(B)** 3000 **(C)** 3500 **(D)** 4000 **(E)** 4500

2009A 6. Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

(A) P^2Q **(B)** P^nQ^m **(C)** P^nQ^{2m} **(D)** $P^{2m}Q^n$ **(E)** $P^{2n}Q^m$

2010A 6. A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

2012A 6. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2015B 6. Back in 1930, Tillie had to memorize her multiplication facts from 0×0 through 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

(A) 0.21 **(B)** 0.25 **(C)** 0.46 **(D)** 0.50 **(E)** 0.75

2004A

7. A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token into a discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

(A) 36 **(B)** 37 **(C)** 38 **(D)** 39 **(E)** 40

2012B

7. Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

(A) 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

2016B

7. Josh writes the numbers $1, 2, 3, \ldots, 99, 100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of his list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

(A) 13 **(B)** 32 **(C)** 56 **(D)** 64 **(E)** 96

2018A

7. For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

1999 9. Before Ashley started a three-hour drive, her car's odometer reading was 29792, a palindrome. (A palindrome is a number that reads the same way from left to right as it does from right to left.) At her destination, the odometer reading was another palindrome. If Ashley never exceeded the speed limit of 75 miles per hour, which of the following was her greatest possible average speed?

(A) $33\frac{1}{3}$ **(B)** $53\frac{1}{3}$ **(C)** $66\frac{2}{3}$ **(D)** $70\frac{1}{3}$ **(E)** $74\frac{1}{3}$

2006B 9. How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?

(A) 21 **(B)** 34 **(C)** 51 **(D)** 72 **(E)** 150

2014A 9. Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

(A) $a+3$ **(B)** $a+4$ **(C)** $a+5$ **(D)** $a+6$ **(E)** $a+7$

2014A 10. Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

(A) $\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

2014B 10. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?

(A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41