UNIT 9 EXERCISES 16-20

SYSTEM OF EQ

2001

21. Four positive integers a, b,c, and d have a product of $8!$ and satisfy

$$ab + a + b = 524,$$
$$bc + b + c = 146, and$$
$$cd + c + d = 104.$$

What is $a - d$?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

2012A

21. Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011 \text{ and}$$
$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

2015B

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

2005B

23. Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x + y) = z \quad \text{and} \quad \log_{10}(x^2 + y^2) = z + 1.$$

There are real numbers a and b such that for all ordered triples (x, y, z) in S we have $x^3 + y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a + b$?

(A) $\frac{15}{2}$ **(B)** $\frac{29}{2}$ **(C)** 15 **(D)** $\frac{39}{2}$ **(E)** 24

2003B 24. Positive integers a, b, and c are chosen so that $a < b < c$, and the system of equations

$$2x + y = 2003 \quad \text{and} \quad y = |x - a| + |x - b| + |x - c|$$

has exactly one solution. What is the minimum value of c?

(A) 668 **(B)** 669 **(C)** 1002 **(D)** 2003 **(E)** 2004