UNIT 6 EXERCISES 16-20

COMBINATIONS

2008A 21. A permutation $(a_1, a_2, a_3, a_4, a_5)$ of (1, 2, 3, 4, 5) is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

(A) 36 **(B)** 40 **(C)** 44 **(D)** 48 **(E)** 52

2009B 21. Ten women sit in 10 seats in a line. All of the 10 get up and then reseat themselves using all 10 seats, each sitting in the seat she was in before or a seat next to the one she occupied before. In how many ways can the women be reseated?

(A) 89 **(B)** 90 **(C)** 120 **(D)** 2^{10} **(E)** $2^2 3^8$

2004B 22. The square

50	b	c
d	e	f
g	h	2

is a multiplicative magic square. That is, the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers, what is the sum of the possible values of g?

(A) 10 **(B)** 25 **(C)** 35 **(D)** 62 **(E)** 136

2009B 22. Parallelogram $ABCD$ has area 1,000,000. Vertex A is at $(0,0)$ and all other vertices are in the first quadrant. Vertices B and D are lattice points on the lines $y = x$ and $y = kx$ for some integer $k > 1$, respectively. How many such parallelograms are there?

(A) 49 **(B)** 720 **(C)** 784 **(D)** 2009 **(E)** 2048

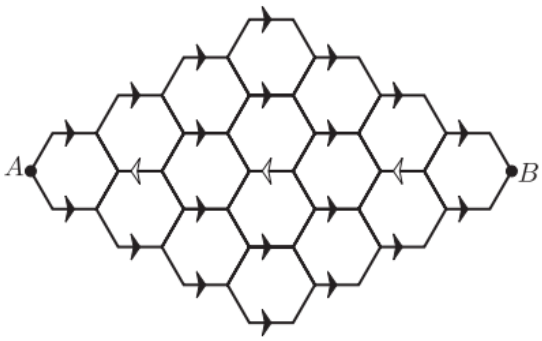
2011A

22. Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called *n-ray partitional* if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

(A) 1500 **(B)** 1560 **(C)** 2320 **(D)** 2480 **(E)** 2500

2012B

22. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

2015B 22. Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

2011B 23. A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

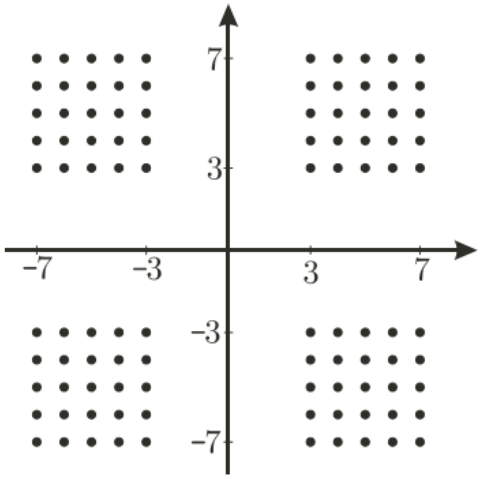
(A) 161 **(B)** 185 **(C)** 195 **(D)** 227 **(E)** 255

2005A 25. Let S be the set of all points with coordinates (x, y, z), where x, y, and z are each chosen from the set $\{0, 1, 2\}$. How many equilateral triangles have all their vertices in S ?

(A) 72 **(B)** 76 **(C)** 80 **(D)** 84 **(E)** 88

2009B

25. The set G is defined by the points (x, y) with integer coordinates, $3 \leq |x| \leq 7$, and $3 \leq |y| \leq 7$. How many squares of side at least 6 have their four vertices in G?



(A) 125 **(B)** 150 **(C)** 175 **(D)** 200 **(E)** 225