UNIT 5 EXERCISES 16-20

GEO WORD

2016B

23. What is the volume of the region in three-dimensional space defined by the inequalities $|x| + |y| + |z| \leq 1$ and $|x| + |y| + |z-1| \leq 1$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

2008B

21. Two circles of radius 1 are to be constructed as follows. The center of circle A is chosen uniformly and at random from the line segment joining $(0,0)$ to $(2,0)$. The center of circle B is chosen uniformly and at random, and independently of the first choice, from the line segment joining $(0,1)$ to $(2,1)$. What is the probability that circles A and B intersect?

(A) $\frac{2+\sqrt{2}}{4}$ **(B)** $\frac{3\sqrt{3}+2}{8}$ **(C)** $\frac{2\sqrt{2}-1}{2}$ **(D)** $\frac{2+\sqrt{3}}{4}$ **(E)** $\frac{4\sqrt{3}-3}{4}$

2015A

21. A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2+16y^2=16$. The set of all possible values of r is an interval $[a,b)$. What is $a+b$?

(A) $5\sqrt{2}+4$ **(B)** $\sqrt{17}+7$ **(C)** $6\sqrt{2}+3$ **(D)** $\sqrt{15}+8$ **(E)** 12

1999

22. The graphs of $y=-|x-a|+b$ and $y=|x-c|+d$ intersect at points $(2,5)$ and $(8,3)$. Find $a+c$.

(A) 7 **(B)** 8 **(C)** 10 **(D)** 13 **(E)** 18

2017A

22. A square is drawn in the Cartesian coordinate plane with vertices at $(2,2)$, $(-2,2)$, $(-2,-2)$, and $(2,-2)$. A particle starts at $(0,0)$. Every second it moves with equal probability to one of the eight lattice points (points with integer coordinates) closest to its current position, independently of its previous moves. In other words, the probability is $\frac{1}{8}$ that the particle will move from (x,y) to each of $(x,y+1)$, $(x+1,y+1)$, $(x+1,y)$, $(x+1,y-1)$, $(x,y-1)$, $(x-1,y-1)$, $(x-1,y)$, or $(x-1,y+1)$. The particle will eventually hit the square for the first time, either at one of the 4 corners of the square or at one of the 12 lattice points in the interior of one of the sides of the square. The probability that it will hit at a corner rather than at an interior point of a side is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 4 **(B)** 5 **(C)** 7 **(D)** 15 **(E)** 39

2005B

24. All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x-coordinates of the three vertices have a sum of m/n, where m and n are relatively prime positive integers. What is the value of $m+n$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18