UNIT 4 EXERCISES 16-20

TRIANGLES

1999

21. A circle is circumscribed about a triangle with sides 20, 21, and 29, thus dividing the interior of the circle into four regions. Let A, B, and C be the areas of the non-triangular regions, with C being the largest. Then

(A) $A+B=C$ **(B)** $A+B+210=C$ **(C)** $A^2+B^2=C^2$

(D) $20A+21B=29C$ **(E)** $\frac{1}{A^2}+\frac{1}{B^2}=\frac{1}{C^2}$

2016B

21. Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i = 1, 2, \ldots,$ let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What is

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i\,?$$

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

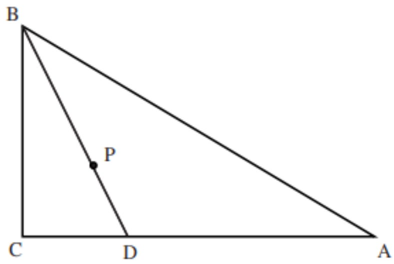
2018B

21. In $\triangle ABC$ with side lengths $AB = 13$, $AC = 12$, and $BC = 5$, let O and I denote the circumcenter and incenter, respectively. A circle with center M is tangent to the legs $\overline{AC}$ and $\overline{BC}$ and to the circumcircle of $\triangle ABC$. What is the area of $\triangle MOI$?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** 3 **(D)** $\frac{13}{4}$ **(E)** $\frac{7}{2}$

2002A

22. Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60°$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



(A) $\frac{2-\sqrt{2}}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3-\sqrt{3}}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5-\sqrt{5}}{5}$

2007B 22. Two particles move along the edges of equilateral $\triangle ABC$ in the direction

$$A \to B \to C \to A,$$

starting simultaneously and moving at the same speed. One starts at A, and the other starts at the midpoint of $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of R to the area of $\triangle ABC$?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

2011B 22. Let T_1 be a triangle with sides 2011, 2012, and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and D, E, and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$

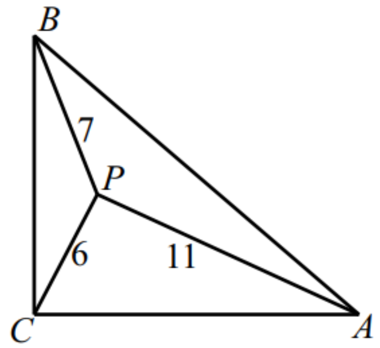
2002A 23. In triangle ABC, side $\overline{AC}$ and the perpendicular bisector of $\overline{BC}$ meet in point D, and $\overline{BD}$ bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

(A) 14 **(B)** 21 **(C)** 28 **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

2002B 23. In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $\overline{BC}$ and the median from A to $\overline{BC}$ have the same length. What is BC?

(A) $\frac{1+\sqrt{2}}{2}$ **(B)** $\frac{1+\sqrt{3}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\sqrt{3}$

2006B 23. Isosceles $\triangle ABC$ has a right angle at C. Point P is inside $\triangle ABC$, such that $PA = 11$, $PB = 7$, and $PC = 6$. Legs $\overline{AC}$ and $\overline{BC}$ have length $s = \sqrt{a + b\sqrt{2}}$, where a and b are positive integers. What is $a + b$?



(A) 85 **(B)** 91 **(C)** 108 **(D)** 121 **(E)** 127

2007B 23. How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

2018A

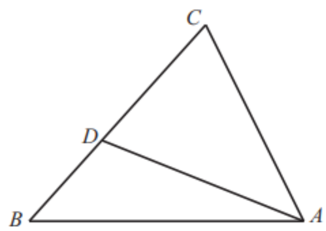
23. In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

(A) 76 **(B)** 77 **(C)** 78 **(D)** 79 **(E)** 80

2001

24. In triangle ABC, $\angle ABC = 45^\circ$. Point D is on $\overline{BC}$ so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$.

(A) 54° **(B)** 60° **(C)** 72° **(D)** 75° **(E)** 90°



2008A

24. Triangle ABC has $\angle C = 60^\circ$ and $BC = 4$. Point D is the midpoint of $\overline{BC}$. What is the largest possible value of $\tan(\angle BAD)$?

(A) $\frac{\sqrt{3}}{6}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{\sqrt{3}}{2\sqrt{2}}$ **(D)** $\frac{\sqrt{3}}{4\sqrt{2}-3}$ **(E)** 1

2008B

24. Let $A_0 = (0, 0)$. Distinct points $A_1, A_2, \dots$ lie on the x-axis, and distinct points $B_1, B_2, \dots$ lie on the graph of $y = \sqrt{x}$. For every positive integer n, $A_{n-1}B_nA_n$ is an equilateral triangle. What is the least n for which the length $A_0A_n \geq 100$?

(A) 13 **(B)** 15 **(C)** 17 **(D)** 19 **(E)** 21

2014A

24. Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?

(A) 299 **(B)** 300 **(C)** 301 **(D)** 302 **(E)** 303

2011A

25. Triangle ABC has $\angle BAC = 60^\circ$, $\angle CBA \leq 90^\circ$, $BC = 1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of the pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

(A) 60° **(B)** 72° **(C)** 75° **(D)** 80° **(E)** 90°