UNIT 3 EXERCISES 16-20

2D GEO WORD PROBLEMS

1999 21. A circle is circumscribed about a triangle with sides $20, 21$, and 29, thus dividing the interior of the circle into four regions. Let A, B, and C be the areas of the non-triangular regions, with C being the largest. Then

(A) $A+B=C$ **(B)** $A+B+210=C$ **(C)** $A^2+B^2=C^2$

(D) $20A+21B=29C$ **(E)** $\frac{1}{A^2}+\frac{1}{B^2}=\frac{1}{C^2}$

2006B 21. Rectangle $ABCD$ has area 2006. An ellipse with area 2006π passes through A and C and has foci at B and D. What is the perimeter of the rectangle? (The area of an ellipse is πab, where $2a$ and $2b$ are the lengths of its axes.)

(A) $\frac{16\sqrt{2006}}{\pi}$ **(B)** $\frac{1003}{4}$ **(C)** $8\sqrt{1003}$ **(D)** $6\sqrt{2006}$ **(E)** $\frac{32\sqrt{1003}}{\pi}$

2016A 21. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

(A) 200 **(B)** $200\sqrt{2}$ **(C)** $200\sqrt{3}$ **(D)** $300\sqrt{2}$ **(E)** 500

2006A 22. A circle of radius r is concentric with and outside a regular hexagon of side length 2. The probability that three entire sides of the hexagon are visible from a randomly chosen point on the circle is $1/2$. What is r?

(A) $2\sqrt{2}+2\sqrt{3}$ **(B)** $3\sqrt{3}+\sqrt{2}$ **(C)** $2\sqrt{6}+\sqrt{3}$ **(D)** $3\sqrt{2}+\sqrt{6}$

(E) $6\sqrt{2}-\sqrt{3}$

2009A 22. A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a+b+c$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

1999 **23. The equiangular convex hexagon $ABCDEF$ has $AB=1, BC=4, CD=2$, and $DE=4$. The area of the hexagon is**

(A) $\frac{15}{2}\sqrt{3}$ **(B)** $9\sqrt{3}$ **(C)** 16 **(D)** $\frac{39}{4}\sqrt{3}$ **(E)** $\frac{43}{4}\sqrt{3}$

2007A 23. Square $ABCD$ has area 36, and $\overline{AB}$ is parallel to the x-axis. Vertices A, B, and C are on the graphs of $y=\log_a x$, $y=2\log_a x$, and $y=3\log_a x$, respectively. What is a?

(A) $\sqrt[6]{3}$ **(B)** $\sqrt{3}$ **(C)** $\sqrt[3]{6}$ **(D)** $\sqrt{6}$ **(E)** 6

2013A 23. $ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP=\sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi+b)$, where a, b, and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

(A) 15 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 23

1999 24. Six points on a circle are given. Four of the chords joining pairs of the six points are selected at random. What is the probability that the four chords form a convex quadrilateral?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{91}$ **(C)** $\frac{1}{273}$ **(D)** $\frac{1}{455}$ **(E)** $\frac{1}{1365}$

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2002B

24. A convex quadrilateral $ABCD$ with area 2002 contains a point P in its interior such that $PA = 24$, $PB = 32$, $PC = 28$, and $PD = 45$. Find the perimeter of $ABCD$.

(A) $4\sqrt{2002}$ **(B)** $2\sqrt{8465}$ **(C)** $2\left(48 + \sqrt{2002}\right)$

(D) $2\sqrt{8633}$ **(E)** $4\left(36 + \sqrt{113}\right)$

2011A

24. Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, and $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

(A) $\sqrt{15}$ **(B)** $\sqrt{21}$ **(C)** $2\sqrt{6}$ **(D)** 5 **(E)** $2\sqrt{7}$

2014B

24. Let $ABCDE$ be a pentagon inscribed in a circle such that $AB = CD = 3$, $BC = DE = 10$, and $AE = 14$. The sum of the lengths of all diagonals of $ABCDE$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 129 **(B)** 247 **(C)** 353 **(D)** 391 **(E)** 421

2015B

24. Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\frac{5}{8}$ times the radius of circle B, and the radius of circle C is $\frac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR + BR + CR + DR$?

(A) 180 **(B)** 184 **(C)** 188 **(D)** 192 **(E)** 196

2017A

24. Quadrilateral $ABCD$ is inscribed in circle O and has sides $AB = 3$, $BC = 2$, $CD = 6$, and $DA = 8$. Let X and Y be points on $\overline{BD}$ such that

$$\frac{DX}{BD} = \frac{1}{4} \quad \text{and} \quad \frac{BY}{BD} = \frac{11}{36}.$$

Let E be the intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF \cdot XG$?

(A) 17 **(B)** $\frac{59 - 5\sqrt{2}}{3}$ **(C)** $\frac{91 - 12\sqrt{3}}{4}$ **(D)** $\frac{67 - 10\sqrt{2}}{3}$

(E) 18

2017B

24. Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\frac{AB}{BC}$?

(A) $1+\sqrt{2}$ **(B)** $2+\sqrt{2}$ **(C)** $\sqrt{17}$ **(D)** $2+\sqrt{5}$ **(E)** $1+2\sqrt{3}$

2003B

25. Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distances between the points are less than the radius of the circle?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{9}$

2007B

25. Points A,B,C,D, and E are located in 3-dimensional space with $AB = BC = CD = DE = EA = 2$ and $\angle ABC = \angle CDE = \angle DEA = 90^\circ$. The plane of $\triangle ABC$ is parallel to $\overline{DE}$. What is the area of $\triangle BDE$?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $\sqrt{5}$ **(E)** $\sqrt{6}$

2008B

25. Let $ABCD$ be a trapezoid with $AB \parallel CD$, $AB = 11$, $BC = 5$, $CD = 19$, and $DA = 7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?

(A) $28\sqrt{3}$ **(B)** $30\sqrt{3}$ **(C)** $32\sqrt{3}$ **(D)** $35\sqrt{3}$ **(E)** $36\sqrt{3}$

2010A

25. Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

(A) 560 **(B)** 564 **(C)** 568 **(D)** 1498 **(E)** 2255

1999

26. Three non-overlapping regular plane polygons, at least two of which are congruent, all have sides of length 1. The polygons meet at a point A in such a way that the sum of the three interior angles at A is $360°$. Thus the three polygons form a new polygon with A as an interior point. What is the largest possible perimeter that this polygon can have?

(A) 12 **(B)** 14 **(C)** 18 **(D)** 21 **(E)** 24