UNIT 2 EXERCISES 16-20

3D GEO

2004A 22. Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

(A) $3+\frac{\sqrt{30}}{2}$ **(B)** $3+\frac{\sqrt{69}}{3}$ **(C)** $3+\frac{\sqrt{123}}{4}$ **(D)** $\frac{52}{9}$ **(E)** $3+2\sqrt{2}$

2005A 22. A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 16

2015B

23. A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

(A) 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

2018B

23. Ajay is standing at point A near Pontianak, Indonesia, 0° latitude and 110° E longitude. Billy is standing at point B near Big Baldy Mountain, Idaho, USA, 45° N latitude and 115° W longitude. Assume that Earth is a perfect sphere with center C. What is the degree measure of $\angle ACB$?

(A) 105 **(B)** $112\frac{1}{2}$ **(C)** 120 **(D)** 135 **(E)** 150

2004A

24. A plane contains points A and B with $AB = 1$. Let S be the union of all disks of radius 1 in the plane that cover $\overline{AB}$. What is the area of S?

(A) $2\pi + \sqrt{3}$ **(B)** $\frac{8\pi}{3}$ **(C)** $3\pi - \frac{\sqrt{3}}{2}$ **(D)** $\frac{10\pi}{3} - \sqrt{3}$ **(E)** $4\pi - 2\sqrt{3}$

1999

25. There are unique integers $a_2, a_3, a_4, a_5, a_6, a_7$ such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

where $0 \le a_i < i$ for $i = 2, 3, \ldots, 7$. Find $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$.

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2015B

25. A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \geq 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j+1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b} + c\sqrt{d}$ inches away from P_0, where a, b, c, and d are positive integers and b and d are not divisible by the square of any prime. What is $a + b + c + d$?

(A) 2016 **(B)** 2024 **(C)** 2032 **(D)** 2040 **(E)** 2048

1999

29. A tetrahedron with four equilateral triangular faces has a sphere inscribed within it and a sphere circumscribed about it. For each of the four faces, there is a sphere tangent externally to the face at its center and to the circumscribed sphere. A point P is selected at random inside the circumscribed sphere. The probability that P lies inside one of the five small spheres is closest to

(A) 0 **(B)** 0.1 **(C)** 0.2 **(D)** 0.3 **(E)** 0.4