UNIT 17 EXERCISES 16-20

TRIG

2004A 21. If $\sum_{n=0}^{\infty} \cos^{2n}\theta = 5$, what is the value of $\cos 2\theta$?

(A) $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{\sqrt{5}}{5}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{4}{5}$

2003B 23. The number of x-intercepts on the graph of $y = \sin(1/x)$ in the interval $(0.0001, 0.001)$ is closest to

(A) 2900 **(B)** 3000 **(C)** 3100 **(D)** 3200 **(E)** 3300

2006B 24. Let S be the set of all points (x, y) in the coordinate plane such that $0 \leq x \leq \frac{\pi}{2}$ and $0 \leq y \leq \frac{\pi}{2}$. What is the area of the subset of S for which

$$\sin^2 x - \sin x \sin y + \sin^2 y \leq \frac{3}{4}\,?$$

(A) $\frac{\pi^2}{9}$ **(B)** $\frac{\pi^2}{8}$ **(C)** $\frac{\pi^2}{6}$ **(D)** $\frac{3\pi^2}{16}$ **(E)** $\frac{2\pi^2}{9}$

2007A

24. For each integer $n > 1$, let $F(n)$ be the number of solutions of the equation $\sin x = \sin nx$ on the interval $[0, \pi]$. What is $\sum_{n=2}^{2007} F(n)$?

(A) 2,014,524 **(B)** 2,015,028 **(C)** 2,015,033 **(D)** 2,016,532

(E) 2,017,033

2009B

24. For how many values of x in $[0, \pi]$ is $\sin^{-1}(\sin 6x) = \cos^{-1}(\cos x)$?

Note: The functions $\sin^{-1} = \arcsin$ and $\cos^{-1} = \arccos$ denote inverse trigonometric functions.

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

2010A

24. Let $f(x) = \log_{10}(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x))$. The intersection of the domain of $f(x)$ with the interval $[0, 1]$ is a union of n disjoint open intervals. What is n ?

(A) 2 **(B)** 12 **(C)** 18 **(D)** 22 **(E)** 36

2015A 24. Rational numbers a and b are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where n and d are integers with $1 \leq d \leq 5$. What is the probability that

$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

(A) $\frac{3}{50}$ **(B)** $\frac{4}{25}$ **(C)** $\frac{41}{200}$ **(D)** $\frac{6}{25}$ **(E)** $\frac{13}{50}$

2014B 25. What is the sum of all positive real solutions x to the equation

$$2\cos(2x)\left(\cos(2x) - \cos\left(\frac{2014\pi^2}{x}\right)\right) = \cos(4x) - 1\,?$$

(A) π **(B)** 810π **(C)** 1008π **(D)** 1080π **(E)** 1800π

1999 27. In triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then $\angle C$ in degrees is

(A) 30 **(B)** 60 **(C)** 90 **(D)** 120 **(E)** 150