UNIT 15 EXERCISES 16-20

POLYNOMIALS

2003A 21. The graph of the polynomial

$$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

has five distinct x-intercepts, one of which is at $(0, 0)$. Which of the following coefficients cannot be zero?

(A) a **(B)** b **(C)** c **(D)** d **(E)** e

2004B 21. The graph of $2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$ is an ellipse in the first quadrant of the xy-plane. Let a and b be the maximum and minimum values of $\frac{y}{x}$ over all points (x, y) on the ellipse. What is the value of $a + b$?

(A) 3 **(B)** $\sqrt{10}$ **(C)** $\frac{7}{2}$ **(D)** $\frac{9}{2}$ **(E)** $2\sqrt{14}$

2007A 21. The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^2 + bx + c$ are equal. Their common value must also be which of the following?

(A) the coefficient of x^2 **(B)** the coefficient of x

(C) the y-intercept of the graph of $y = f(x)$

(D) one of the x-intercepts of the graph of $y = f(x)$

(E) the mean of the x-intercepts of the graph of $y = f(x)$

2010A 21. The graph of $y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$ lies above the line $y = bx + c$ except at three values of x, where the graph and the line intersect. What is the largest of those values?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2010B 21. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$
$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

2014A 21. For every real number x, let $\lfloor x \rfloor$ denote the greatest integer not exceeding x, and let

$$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1).$$

The set of all numbers x such that $1 \leq x < 2014$ and $f(x) \leq 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

(A) 1 **(B)** $\frac{\log 2015}{\log 2014}$ **(C)** $\frac{\log 2014}{\log 2013}$ **(D)** $\frac{2014}{2013}$ **(E)** $2014^{\frac{1}{2014}}$

2018A 21. Which of the following polynomials has the greatest real root?

(A) $x^{19} + 2018x^{11} + 1$ **(B)** $x^{17} + 2018x^{11} + 1$

(C) $x^{19} + 2018x^{13} + 1$ **(D)** $x^{17} + 2018x^{13} + 1$

(E) $2019x + 2018$

1999 22. The graphs of $y = -|x - a| + b$ and $y = |x - c| + d$ intersect at points $(2, 5)$ and $(8, 3)$. Find $a + c$.

(A) 7 **(B)** 8 **(C)** 10 **(D)** 13 **(E)** 18

2010A 22. What is the minimum value of $f(x) = |x-1|+|2x-1|+|3x-1|+\cdots+|119x-1|$?

(A) 49 **(B)** 50 **(C)** 51 **(D)** 52 **(E)** 53

2018B 22. Consider polynomials $P(x)$ of degree at most 3, each of whose coefficients is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many such polynomials satisfy $P(-1) = -9$?

(A) 110 **(B)** 143 **(C)** 165 **(D)** 220 **(E)** 286

2004B 23. The polynomial $x^3 - 2004x^2 + mx + n$ has integer coefficients and three distinct positive zeros. Exactly one of these is an integer, and it is the sum of the other two. How many values of n are possible?

(A) 250,000 **(B)** 250,250 **(C)** 250,500 **(D)** 250,750 **(E)** 251,000

2010B 23. Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeros at $x = -23, -21, -17$, and -15, and $Q(P(x))$ has zeros at $x = -59, -57, -51$, and -49. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

(A) -100 **(B)** -82 **(C)** -73 **(D)** -64 **(E)** 0

2017A 23. For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

(A) -9009 **(B)** -8008 **(C)** -7007 **(D)** -6006 **(E)** -5005

2017B

23. The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** $\frac{24}{5}$ **(E)** 8

2005A

24. Let $P(x) = (x-1)(x-2)(x-3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?

(A) 19 **(B)** 22 **(C)** 24 **(D)** 27 **(E)** 32

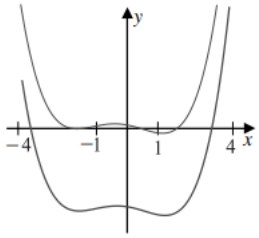
2016A

24. There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a the value of b is unique. What is this value of b?

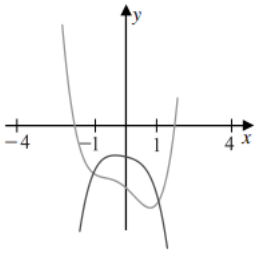
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2002A 25. The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \leq x \leq 4$?

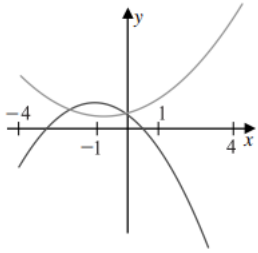
(A)



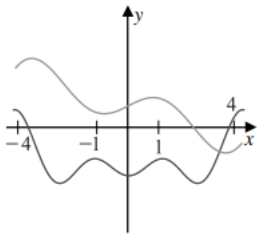
(B)



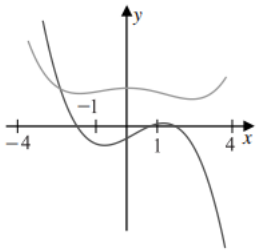
(C)



(D)



(E)



2014A 25. The parabola P has focus $(0, 0)$ and goes through the points $(4, 3)$ and $(-4, -3)$. For how many points $(x, y) \in P$ with integer coordinates is it true that $|4x + 3y| \leq 1000$?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46