

## UNIT 5 QUESTIONS 16-20

## CO-ORD GEO

- 2006B 16. Regular hexagon  $ABCDEF$  has vertices  $A$  and  $C$  at  $(0,0)$  and  $(7,1)$ , respectively. What is its area?
- (A)  $20\sqrt{3}$       (B)  $22\sqrt{3}$       (C)  $25\sqrt{3}$       (D)  $27\sqrt{3}$       (E) 50
- 2016A 16. The graphs of  $y = \log_3 x$ ,  $y = \log_x 3$ ,  $y = \log_{\frac{1}{3}} x$ , and  $y = \log_x \frac{1}{3}$  are plotted on the same set of axes. How many points in the plane with positive  $x$ -coordinates lie on two or more of the graphs?
- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6
- 2018A 16. Which of the following describes the set of values of  $a$  for which the curves  $x^2 + y^2 = a^2$  and  $y = x^2 - a$  in the real  $xy$ -plane intersect at exactly 3 points?
- (A)  $a = \frac{1}{4}$       (B)  $\frac{1}{4} < a < \frac{1}{2}$       (C)  $a > \frac{1}{4}$       (D)  $a = \frac{1}{2}$
- (E)  $a > \frac{1}{2}$

- 2008B 17. Let  $A$ ,  $B$  and  $C$  be three distinct points on the graph of  $y = x^2$  such that line  $AB$  is parallel to the  $x$ -axis and  $\triangle ABC$  is a right triangle with area 2008. What is the sum of the digits of the  $y$ -coordinate of  $C$ ?
- (A) 16      (B) 17      (C) 18      (D) 19      (E) 20

- 2012B 17. Square  $PQRS$  lies in the first quadrant. Points  $(3, 0)$ ,  $(5, 0)$ ,  $(7, 0)$ , and  $(13, 0)$  lie on lines  $SP$ ,  $RQ$ ,  $PQ$ , and  $SR$ , respectively. What is the sum of the coordinates of the center of the square  $PQRS$ ?
- (A) 6      (B) 6.2      (C) 6.4      (D) 6.6      (E) 6.8

- 2014B 17. Let  $\mathcal{P}$  be the parabola with equation  $y = x^2$  and let  $Q = (20, 14)$ . There are real numbers  $r$  and  $s$  such that the line through  $Q$  with slope  $m$  does not intersect  $\mathcal{P}$  if and only if  $r < m < s$ . What is  $r + s$ ?
- (A) 1      (B) 26      (C) 40      (D) 52      (E) 80

- 2002A 18. Let  $C_1$  and  $C_2$  be circles defined by

$$(x - 10)^2 + y^2 = 36$$

and

$$(x + 15)^2 + y^2 = 81,$$

respectively. What is the length of the shortest line segment  $\overline{PQ}$  that is tangent to  $C_1$  at  $P$  and to  $C_2$  at  $Q$ ?

- (A) 15      (B) 18      (C) 20      (D) 21      (E) 24

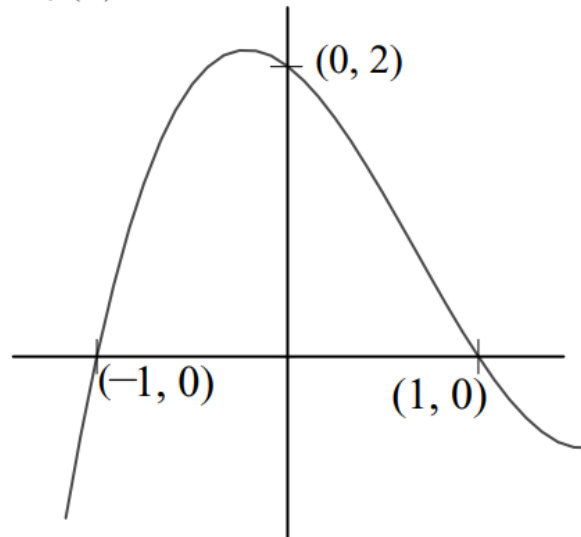
- 2005B 18. Let  $A(2, 2)$  and  $B(7, 7)$  be points in the plane. Define  $R$  as the region in the first quadrant consisting of those points  $C$  such that  $\triangle ABC$  is an acute triangle. What is the closest integer to the area of the region  $R$ ?
- (A) 25                      (B) 39                      (C) 51                      (D) 60                      (E) 80

- 2016B 18. What is the area of the region enclosed by the graph of the equation  $x^2 + y^2 = |x| + |y|$ ?
- (A)  $\pi + \sqrt{2}$       (B)  $\pi + 2$       (C)  $\pi + 2\sqrt{2}$       (D)  $2\pi + \sqrt{2}$       (E)  $2\pi + 2\sqrt{2}$

- 2011B 19. A lattice point in an  $xy$ -coordinate system is any point  $(x, y)$  where both  $x$  and  $y$  are integers. The graph of  $y = mx + 2$  passes through no lattice point with  $0 < x \leq 100$  for all  $m$  such that  $\frac{1}{2} < m < a$ . What is the maximum possible value of  $a$ ?
- (A)  $\frac{51}{101}$       (B)  $\frac{50}{99}$       (C)  $\frac{51}{100}$       (D)  $\frac{52}{101}$       (E)  $\frac{13}{25}$

2011

- 2003B 20. Part of the graph of  $f(x) = ax^3 + bx^2 + cx + d$  is shown. What is  $b$ ?



- (A)  $-4$       (B)  $-2$       (C)  $0$       (D)  $2$       (E)  $4$

- 2007B 20. The parallelogram bounded by the lines  $y = ax + c$ ,  $y = ax + d$ ,  $y = bx + c$ , and  $y = bx + d$  has area 18. The parallelogram bounded by the lines  $y = ax + c$ ,  $y = ax - d$ ,  $y = bx + c$ , and  $y = bx - d$  has area 72. Given that  $a$ ,  $b$ ,  $c$ , and  $d$  are positive integers, what is the smallest possible value of  $a + b + c + d$ ?

- (A) 13      (B) 14      (C) 15      (D) 16      (E) 17