UNIT 4 QUESTIONS 16-20

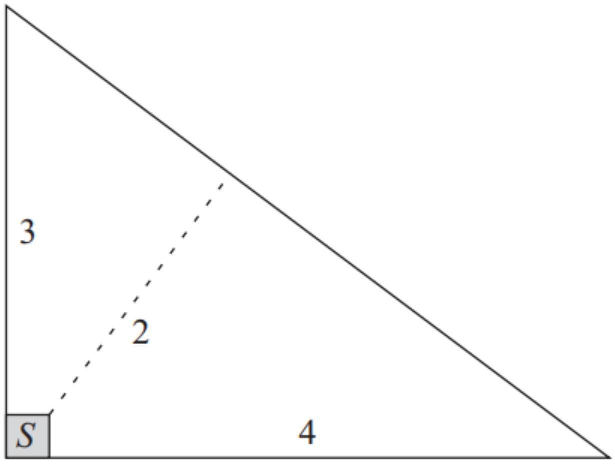
TRIANGLES

2010A 17. Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

2018A

17. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths of 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

2012A

18. Triangle ABC has $AB = 27$, $AC = 26$, and $BC = 25$. Let I denote the intersection of the internal angle bisectors of $\triangle ABC$. What is BI?

(A) 15 **(B)** $5 + \sqrt{26} + 3\sqrt{3}$ **(C)** $3\sqrt{26}$ **(D)** $\frac{2}{3}\sqrt{546}$ **(E)** $9\sqrt{3}$

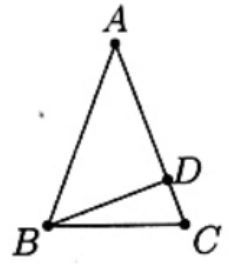
2018A

18. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

1999

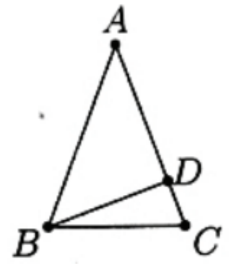
19. Consider all triangles ABC satisfying the following conditions: $AB = AC$, D is a point on $\overline{AC}$ for which $\overline{BD} \perp \overline{AC}$, AD and CD are integers, and $BD^2 = 57$. Among all such triangles, the smallest possible value of AC is



(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

1999

19. Consider all triangles ABC satisfying the following conditions: $AB = AC$, D is a point on $\overline{AC}$ for which $\overline{BD} \perp \overline{AC}$, AD and CD are integers, and $BD^2 = 57$. Among all such triangles, the smallest possible value of AC is



(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

2007A

19. Triangles ABC and ADE have areas 2007 and 7002, respectively, with $B = (0,0)$, $C = (223,0)$, $D = (680,380)$, and $E = (689,389)$. What is the sum of all possible x-coordinates of A?

(A) 282 **(B)** 300 **(C)** 600 **(D)** 900 **(E)** 1200

2013A

19. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

(A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

2014A 19. There are exactly N distinct rational numbers k such that $|k| < 200$ and

$$5x^2 + kx + 12 = 0$$

has at least one integer solution for x. What is N?

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 78

2015B 19. In $\triangle ABC$, $\angle C = 90°$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12 + 9\sqrt{3}$ **(B)** $18 + 6\sqrt{3}$ **(C)** $12 + 12\sqrt{2}$ **(D)** 30 **(E)** 32

2002B 20. Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90°$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

2008A 20. Triangle ABC has $AC = 3$, $BC = 4$, and $AB = 5$. Point D is on $\overline{AB}$, and $\overline{CD}$ bisects the right angle. The inscribed circles of $\triangle ADC$ and $\triangle BCD$ have radii r_a and r_b, respectively. What is r_a/r_b?

(A) $\frac{1}{28}(10 - \sqrt{2})$ **(B)** $\frac{3}{56}(10 - \sqrt{2})$ **(C)** $\frac{1}{14}(10 - \sqrt{2})$ **(D)** $\frac{5}{56}(10 - \sqrt{2})$ **(E)** $\frac{3}{28}(10 - \sqrt{2})$

2009A 20. Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

(A) $\frac{9}{2}$ **(B)** $\frac{50}{11}$ **(C)** $\frac{21}{4}$ **(D)** $\frac{17}{3}$ **(E)** 6

2011B

20. Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of $\triangle BDE$ and $\triangle CEF$. What is $XA + XB + XC$?

(A) 24 **(B)** $14\sqrt{3}$ **(C)** $\frac{195}{8}$ **(D)** $\frac{129\sqrt{7}}{14}$ **(E)** $\frac{69\sqrt{2}}{4}$

2015A

20. Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of 5, 5, and 8, while those of T' have lengths a, a, and b. Which of the following numbers is closest to b?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

2018A

20. Triangle ABC is an isosceles right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI > AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a + b + c$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

2018B

20. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$