UNIT 22 QUESTIONS 16-20

COMPLEX

2004B 16. A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

2018B 16. The solutions to the equation $(z+6)^8 = 81$ are connected in the complex plane to form a convex regular polygon, three of whose vertices are labeled A, B, and C. What is the least possible area of $\triangle ABC$?

(A) $\frac{1}{6}\sqrt{6}$ **(B)** $\frac{3}{2}\sqrt{2} - \frac{3}{2}$ **(C)** $2\sqrt{3} - 2\sqrt{2}$ **(D)** $\frac{1}{2}\sqrt{2}$

(E) $\sqrt{3} - 1$

2017A 17. There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 24

2008B 19. A function f is defined by $f(z) = (4+i)z^2 + \alpha z + \gamma$ for all complex numbers z, where α and γ are complex numbers and $i^2 = -1$. Suppose that $f(1)$ and $f(i)$ are both real. What is the smallest possible value of $|\alpha| + |\gamma|$?

(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** 4