UNIT 2 QUESTIONS 16-20

3D GEO

2005B 16. Eight spheres of radius 1, one per octant, are each tangent to the coordinate planes. What is the radius of the smallest sphere, centered at the origin, that contains these eight spheres?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $1+\sqrt{2}$ **(D)** $1+\sqrt{3}$ **(E)** 3

2015A 16. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

2015B 16. A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

(A) 18 **(B)** 162 **(C)** $36\sqrt{21}$ **(D)** $18\sqrt{138}$ **(E)** $54\sqrt{21}$

2005A 17. A unit cube is cut twice to form three triangular prisms, two of which are congruent, as shown in Figure 1. The cube is then cut in the same manner along the dashed lines shown in Figure 2. This creates nine pieces. What is the volume of the piece that contains vertex W?

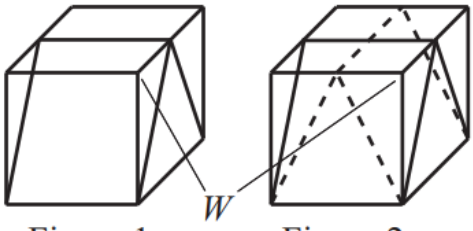


Figure 1 Figure 2

(A) $\frac{1}{12}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

2009B 17. Each face of a cube is given a single narrow stripe painted from the center of one edge to the center of its opposite edge. The choice of the edge pairing is made at random and independently for each face. What is the probability that there is a continuous stripe encircling the cube?

(A) $1/8$ **(B)** $3/16$ **(C)** $1/4$ **(D)** $3/8$ **(E)** $1/2$

2008A 18. Triangle ABC, with sides of length 5, 6, and 7, has one vertex on the positive x-axis, one on the positive y-axis, and one on the positive z-axis. Let O be the origin. What is the volume of tetrahedron $OABC$?

(A) $\sqrt{85}$ **(B)** $\sqrt{90}$ **(C)** $\sqrt{95}$ **(D)** 10 **(E)** $\sqrt{105}$

2008B 18. A pyramid has a square base $ABCD$ and vertex E. The area of square $ABCD$ is 196, and the areas of $\triangle ABE$ and $\triangle CDE$ are 105 and 91, respectively. What is the volume of the pyramid?

(A) 392 **(B)** $196\sqrt{6}$ **(C)** $392\sqrt{2}$ **(D)** $392\sqrt{3}$ **(E)** 784

2011B 18. A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

(A) $5\sqrt{2}-7$ **(B)** $7-4\sqrt{3}$ **(C)** $\frac{2\sqrt{2}}{27}$ **(D)** $\frac{\sqrt{2}}{9}$ **(E)** $\frac{\sqrt{3}}{9}$

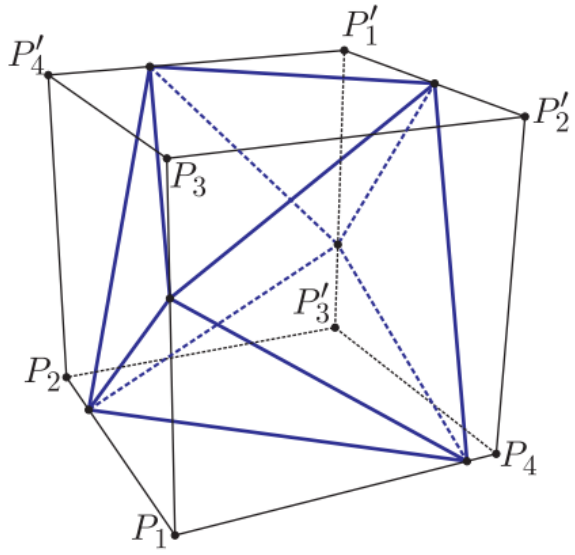
2004B

19. A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?

(A) 6 **(B)** $4\sqrt{5}$ **(C)** 9 **(D)** 10 **(E)** $6\sqrt{3}$

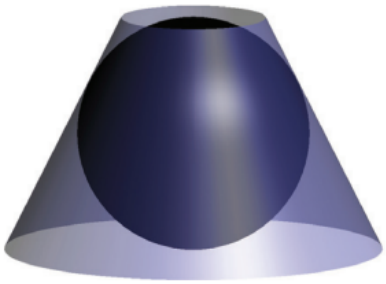
2012B

19. A unit cube has vertices P_1, P_2, P_3, P_4, P_1', P_2', P_3', and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \leq i \leq 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments P_1P_2, P_1P_3, P_1P_4, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?



(A) $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{7\sqrt{6}}{16}$ **(C)** $\frac{\sqrt{5}}{2}$ **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\frac{\sqrt{6}}{2}$

2014B 19. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



(A) $\frac{3}{2}$ **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3+\sqrt{5}}{2}$

2006A 20. A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{1}{2187}$ **(B)** $\frac{1}{729}$ **(C)** $\frac{2}{243}$ **(D)** $\frac{1}{81}$ **(E)** $\frac{5}{243}$

2007A 20. Corners are sliced off a unit cube so that the six faces each become regular octagons. What is the total volume of the removed tetrahedra?

(A) $\frac{5\sqrt{2}-7}{3}$ **(B)** $\frac{10-7\sqrt{2}}{3}$ **(C)** $\frac{3-2\sqrt{2}}{3}$ **(D)** $\frac{8\sqrt{2}-11}{3}$

(E) $\frac{6-4\sqrt{2}}{3}$

2009B 20. A convex polyhedron Q has vertices V_1, V_2, $\ldots$, V_n, and 100 edges. The polyhedron is cut by planes P_1, P_2, $\ldots$, P_n in such a way that plane P_k cuts only those edges that meet at vertex V_k. In addition, no two planes intersect inside or on Q. The cuts produce n pyramids and a new polyhedron R. How many edges does R have?

(A) 200 **(B)** $2n$ **(C)** 300 **(D)** 400 **(E)** $4n$