UNIT 19 QUESTIONS 16-20

QUADRATICS

2014B 16. Let P be a cubic polynomial with $P(0) = k$, $P(1) = 2k$, and $P(-1) = 3k$. What is $P(2) + P(-2)$?

(A) 0 **(B)** k **(C)** $6k$ **(D)** $7k$ **(E)** $14k$

1999 17. Let $P(x)$ be a polynomial such that when $P(x)$ is divided by $x - 19$, the remainder is 99, and when $P(x)$ is divided by $x - 99$, the remainder is 19. What is the remainder when $P(x)$ is divided by $(x - 19)(x - 99)$?

(A) $-x + 80$ **(B)** $x + 80$ **(C)** $-x + 118$ **(D)** $x + 118$ **(E)** 0

2004B 17. For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a?

(A) -256 **(B)** -64 **(C)** -8 **(D)** 64 **(E)** 256

2004B 18. Points A and B are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of $\overline{AB}$. What is the length of $\overline{AB}$?

(A) $2\sqrt{5}$ **(B)** $5 + \frac{\sqrt{2}}{2}$ **(C)** $5 + \sqrt{2}$ **(D)** 7 **(E)** $5\sqrt{2}$

2007A 18. The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2 + i) = 0$. What is $a + b + c + d$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 9 **(E)** 16

2011A 18. Suppose that $|x + y| + |x - y| = 2$. What is the maximum possible value of $x^2 - 6x + y^2$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2015A 18. The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

2003A 19. A parabola with equation $y = ax^2 + bx + c$ is reflected about the x-axis. The parabola and its reflection are translated horizontally five units in opposite directions to become the graphs of $y = f(x)$ and $y = g(x)$, respectively. Which of the following describes the graph of $y = (f + g)(x)$?

(A) a parabola tangent to the x-axis

(B) a parabola not tangent to the x-axis **(C)** a horizontal line

(D) a non-horizontal line **(E)** the graph of a cubic function

2001 19. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2, what is b?

(A) -11 **(B)** -10 **(C)** -9 **(D)** 1 **(E)** 5

2009B 19. For each positive integer n, let $f(n) = n^4 - 360n^2 + 400$. What is the sum of all values of $f(n)$ that are prime numbers?

(A) 794 **(B)** 796 **(C)** 798 **(D)** 800 **(E)** 802

2011A 20. Let $f(x) = ax^2 + bx + c$, where a, b, and c are integers. Suppose that $f(1) = 0$, $50 < f(7) < 60$, $70 < f(8) < 80$, and $5000k < f(100) < 5000(k + 1)$ for some integer k. What is k?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2012A 20. Consider the polynomial

$$P(x) = \prod_{k=0}^{10} (x^{2^k} + 2^k) = (x + 1)(x^2 + 2)(x^4 + 4) \cdots (x^{1024} + 1024).$$

The coefficient of x^{2012} is equal to 2^a. What is a?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 24