UNIT 16 QUESTIONS 16-20

ALGEBRA

2002A

17. Several sets of prime numbers, such as $\{7, 83, 421, 659\}$, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

(A) 193 **(B)** 207 **(C)** 225 **(D)** 252 **(E)** 477

2005B

17. How many distinct four-tuples (a, b, c, d) of rational numbers are there with

$$a\log_{10} 2 + b\log_{10} 3 + c\log_{10} 5 + d\log_{10} 7 = 2005?$$

(A) 0 **(B)** 1 **(C)** 17 **(D)** 2004 **(E)** infinitely many

1999 18. How many zeros does $f(x) = \cos(\log(x))$ have on the interval $0 < x < 1$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** infinitely many

2005A 18. Call a number "prime-looking" if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

(A) 100 **(B)** 102 **(C)** 104 **(D)** 106 **(E)** 108

2009A 18. For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

2012B 18. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

2005A 19. A faulty car odometer proceeds from digit 3 to digit 5 , always skipping the digit 4, regardless of position. For example, after traveling one mile the odometer changed from 000039 to 000050. If the odometer now reads 002005, how many miles has the car actually traveled?

(A) 1404 **(B)** 1462 **(C)** 1604 **(D)** 1605 **(E)** 1804

2017B

19. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

(A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

2012B

20. A trapezoid has side lengths 3, 5, 7, and 11. The sum of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of a prime. What is the greatest integer less than or equal to

$$r_1 + r_2 + r_3 + n_1 + n_2\,?$$

(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65