UNIT 12 QUESTIONS 16-20

ALGEBRA

2004A 16. The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined is $\{x \mid x > c\}$. What is the value of c?

(A) 0 **(B)** 2001^{2002} **(C)** 2002^{2003} **(D)** 2003^{2004} **(E)** $2001^{2002^{2003}}$

2003B 17. If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** 1

2003B 18. Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization $a^c b^d$. What is $a + b + c + d$?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

2002B 19. If a,b, and c are positive real numbers such that $a(b+c) = 152$, $b(c+a) = 162$, and $c(a+b) = 170$, then abc is

(A) 672 **(B)** 688 **(C)** 704 **(D)** 720 **(E)** 750

2008A 19. In the expansion of

$$\left(1 + x + x^2 + \cdots + x^{27}\right)\left(1 + x + x^2 + \cdots + x^{14}\right)^2,$$

what is the coefficient of x^{28} ?

(A) 195 **(B)** 196 **(C)** 224 **(D)** 378 **(E)** 405

2011A 19. At a competition with N players, the number of players given elite status is equal to

$$2^{1+\lfloor \log_2(N-1) \rfloor} - N.$$

Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N ?

Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

(A) 38 **(B)** 90 **(C)** 154 **(D)** 406 **(E)** 1024

1999 20. The sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 19$, $a_9 = 99$, and, for all $n \geq 3$, a_n is the arithmetic mean of the first $n-1$ terms. Find a_2.

(A) 29 **(B)** 59 **(C)** 79 **(D)** 99 **(E)** 179

2002A 20. Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

2005B

20. Let a, b, c, d, e, f, g and h be distinct elements in the set

$$\{-7, -5, -3, -2, 2, 4, 6, 13\}.$$

What is the minimum possible value of

$$(a+b+c+d)^2 + (e+f+g+h)^2?$$

(A) 30 **(B)** 32 **(C)** 34 **(D)** 40 **(E)** 50

2014B

20. For how many positive integers x is $\log_{10}(x-40) + \log_{10}(60-x) < 2$?

(A) 10 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** infinitely many