UNIT 10 QUESTIONS 16-20

PROBABILITY

2002A

16. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) $2/5$ **(B)** $9/20$ **(C)** $1/2$ **(D)** $11/20$ **(E)** $24/25$

2002B

16. Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

2010A

16. Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

2010B

16. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

2017B

16. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over 60,000 positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

(A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

2006B

17. For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio $1:2:3:4:5:6$. What is the probability of rolling a total of 7 on the two dice?

(A) $\frac{4}{63}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{8}{63}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{2}{7}$

2015A

17. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\frac{47}{256}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{49}{256}$ **(D)** $\frac{25}{128}$ **(E)** $\frac{51}{256}$

2015B

17. An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n?

(A) 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

2017B

17. A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

(A) The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

(B) The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

(C) The probabilities are the same.

(D) The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

(E) The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

2002B

18. A point P is randomly selected from the rectangular region with vertices $(0,0), (2,0), (2,1), (0,1)$. What is the probability that P is closer to the origin than it is to the point $(3,1)$?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** 1

2010B

18. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently and at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

2010A

19. Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the k^{th} position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

(A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

2016A

19. Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$? (For example, he succeeds if his sequence of tosses is HTHHHHHH.)

(A) 69 **(B)** 151 **(C)** 257 **(D)** 293 **(E)** 313

2016B

19. Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{3}$

2004A

20. Select numbers a and b between 0 and 1 independently and at random, and let c be their sum. Let A, B, and C be the results when a, b, and c, respectively, are rounded to the nearest integer. What is the probability that $A + B = C$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2004B 20. Each face of a cube is painted either red or blue, each with probability 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$ **(B)** $\frac{5}{16}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

2006B 20. Let x be chosen at random from the interval $(0,1)$. What is the probability that

$$\lfloor \log_{10} 4x \rfloor - \lfloor \log_{10} x \rfloor = 0\,?$$

Here $\lfloor x \rfloor$ denotes the greatest integer that is less than or equal to x.

(A) $\frac{1}{8}$ **(B)** $\frac{3}{20}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

2017B 20. Real numbers x and y are chosen independently and uniformly at random from the interval $(0,1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$