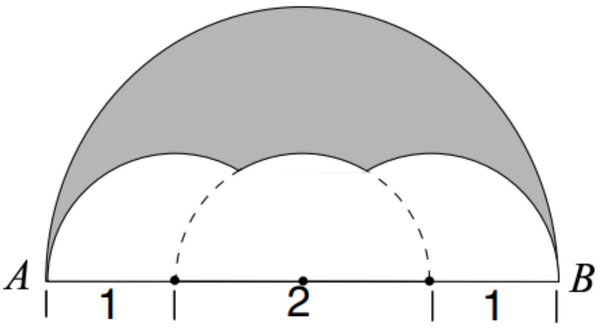
UNIT 1 EXERCISES 16-20

2D GEOMETRY

2003B 16. Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?

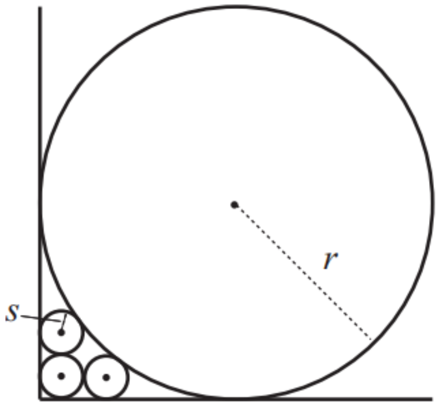


(A) $\pi - \sqrt{3}$ **(B)** $\pi - \sqrt{2}$ **(C)** $\frac{\pi + \sqrt{2}}{2}$ **(D)** $\frac{\pi + \sqrt{3}}{2}$

(E) $\frac{7}{6}\pi - \frac{\sqrt{3}}{2}$

2005A

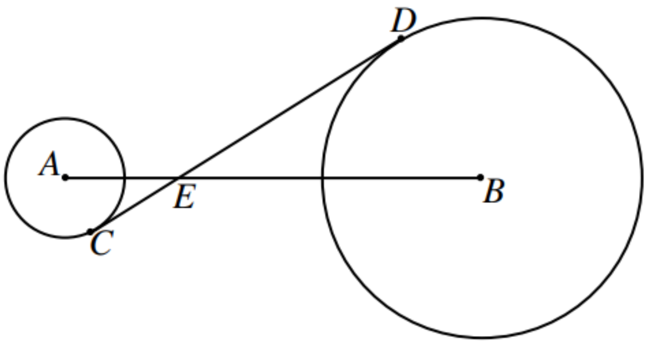
16. Three circles of radius s are drawn in the first quadrant of the xy-plane. The first circle is tangent to both axes, the second is tangent to the first circle and the x-axis, and the third is tangent to the first circle and the y-axis. A circle of radius $r > s$ is tangent to both axes and to the second and third circles. What is r/s?



(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

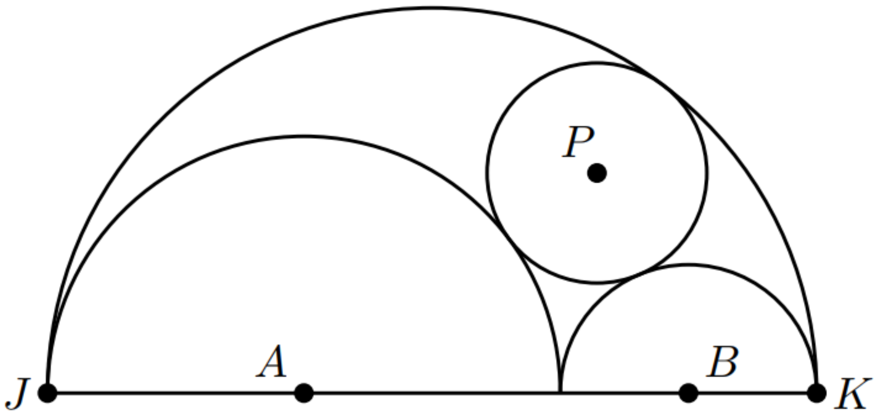
2006A

16. Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent intersects the circles at C and D, respectively. Lines AB and CD intersect at E, and $AE = 5$. What is CD?



(A) 13 **(B)** $\frac{44}{3}$ **(C)** $\sqrt{221}$ **(D)** $\sqrt{255}$ **(E)** $\frac{55}{3}$

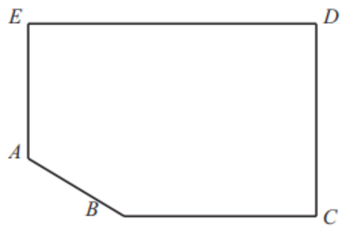
2017A 16. In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter $\overline{JK}$. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?



(A) $\frac{3}{4}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{1}{2}\sqrt{3}$ **(D)** $\frac{5}{8}\sqrt{2}$ **(E)** $\frac{11}{12}$

2001

17. A point P is selected at random from the interior of the pentagon with vertices $A = (0, 2)$, $B = (4, 0)$, $C = (2\pi + 1, 0)$, $D = (2\pi + 1, 4)$, and $E = (0, 4)$. What is the probability that $\angle APB$ is obtuse?

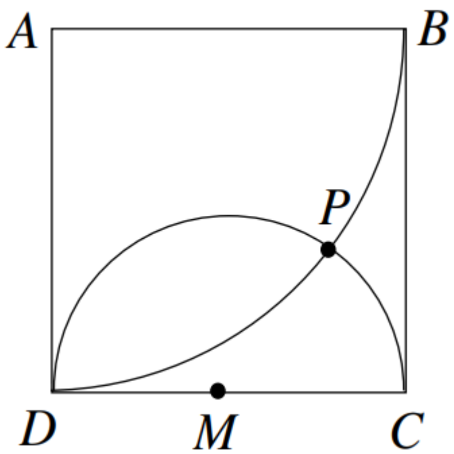


(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{5}{16}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{2}$

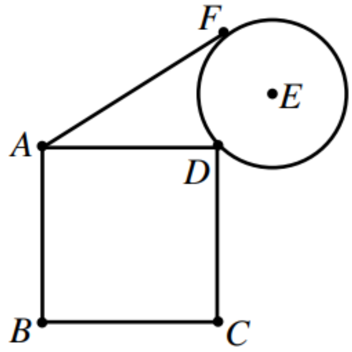
2003A

17. Square $ABCD$ has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects a circle with radius 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?

(A) 3 **(B)** $\frac{16}{5}$ **(C)** $\frac{13}{4}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{7}{2}$

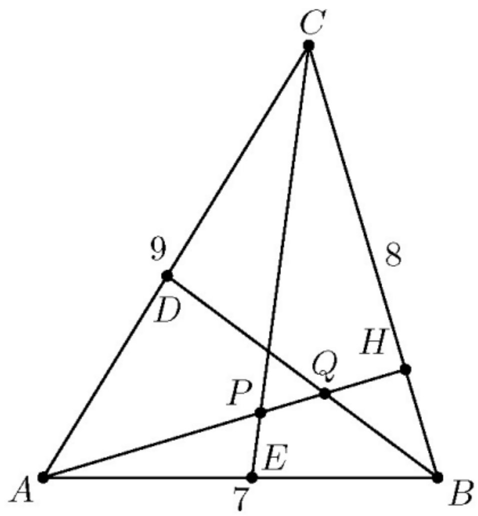


2006A 17. Square $ABCD$ has side length s, a circle centered at E has radius r, and r and s are both rational. The circle passes through D, and D lies on $\overline{BE}$. Point F lies on the circle, on the same side of $\overline{BE}$ as A. Segment AF is tangent to the circle, and $AF = \sqrt{9+5\sqrt{2}}$. What is r/s?



(A) $\frac{1}{2}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{5}{3}$ **(E)** $\frac{9}{5}$

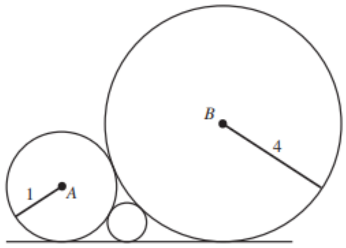
2016B 17. In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?



(A) 1 **(B)** $\frac{5}{8}\sqrt{3}$ **(C)** $\frac{4}{5}\sqrt{2}$ **(D)** $\frac{8}{15}\sqrt{5}$ **(E)** $\frac{6}{5}$

2001

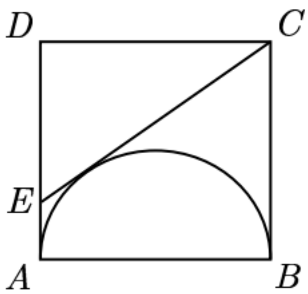
18. A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is



(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{1}{2}$

2004A

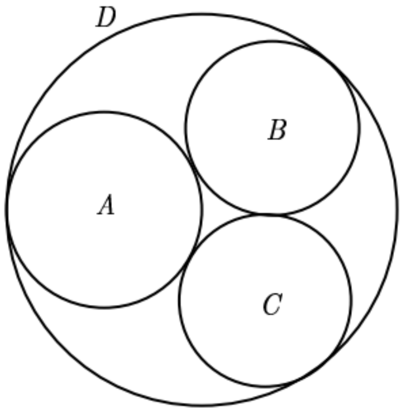
18. Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



(A) $\frac{2+\sqrt{5}}{2}$ **(B)** $\sqrt{5}$ **(C)** $\sqrt{6}$ **(D)** $\frac{5}{2}$ **(E)** $5-\sqrt{5}$

2004A

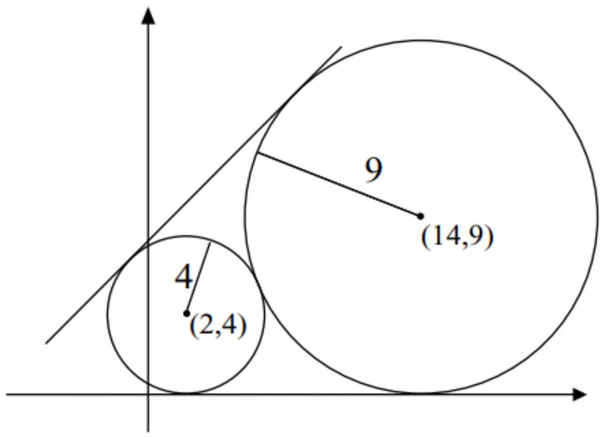
19. Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



(A) $\frac{2}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{1+\sqrt{3}}{3}$

2006A

19. Circles with centers $(2,4)$ and $(14,9)$ have radii 4 and 9, respectively. The equation of a common external tangent to the circles can be written in the form $y = mx + b$ with $m > 0$. What is b?



(A) $\frac{908}{119}$ **(B)** $\frac{909}{119}$ **(C)** $\frac{130}{17}$ **(D)** $\frac{911}{119}$ **(E)** $\frac{912}{119}$