UNIT 4 EXERCISES 11-15

TRIANGLES

2013A 12. The angles in a particular triangle are in arithmetic progression, and the side lengths are 4, 5, and x. The sum of the possible values of x equals $a+\sqrt{b}+\sqrt{c}$, where a, b, and c are positive integers. What is $a+b+c$?

(A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

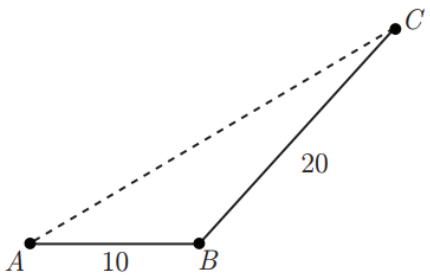
2014B 12. A set S consists of triangles whose sides have integer lengths less than 5, and no two elements of S are congruent or similar. What is the largest number of elements that S can have?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2018B 12. Side $\overline{AB}$ of $\triangle ABC$ has length 10. The bisector of angle A meets $\overline{BC}$ at D, and $CD = 3$. The set of all possible values of AC is an open interval (m, n). What is $m + n$?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

2009A 13. A ship sails 10 miles in a straight line from A to B, turns through an angle between $45°$ and $60°$, and then sails another 20 miles to C. Let AC be measured in miles. Which of the following intervals contains AC^2?



(A) $[400, 500]$ **(B)** $[500, 600]$ **(C)** $[600, 700]$ **(D)** $[700, 800]$
(E) $[800, 900]$

2008B 13. Vertex E of equilateral $\triangle ABE$ is in the interior of unit square $ABCD$. Let R be the region consisting of all points inside $ABCD$ and outside $\triangle ABE$ whose distance from $\overline{AD}$ is between $\frac{1}{3}$ and $\frac{2}{3}$. What is the area of R?

(A) $\frac{12-5\sqrt{3}}{72}$ **(B)** $\frac{12-5\sqrt{3}}{36}$ **(C)** $\frac{\sqrt{3}}{18}$ **(D)** $\frac{3-\sqrt{3}}{9}$ **(E)** $\frac{\sqrt{3}}{12}$

2009B 13. Triangle ABC has $AB = 13$ and $AC = 15$, and the altitude to $\overline{BC}$ has length 12. What is the sum of the two possible values of BC?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

2011A 13. Triangle ABC has side-lengths $AB = 12$, $BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 36 **(E)** 42

2014A 13. A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

(A) 2100 **(B)** 2220 **(C)** 3000 **(D)** 3120 **(E)** 3125

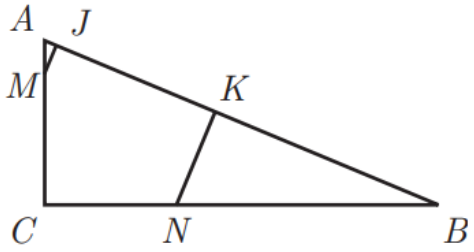
2014B 13. Real numbers a and b are chosen with $1 < a < b$ such that no triangle with positive area has side lengths 1, a, and b or $\frac{1}{b}$, $\frac{1}{a}$, and 1. What is the smallest possible value of b?

(A) $\frac{3+\sqrt{3}}{2}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{3+\sqrt{5}}{2}$ **(D)** $\frac{3+\sqrt{6}}{2}$ **(E)** 3

2016B 13. Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

(A) 3.5 **(B)** 4 **(C)** 4.5 **(D)** 5 **(E)** 5.5

2004B 14. In $\triangle ABC$, $AB = 13$, $AC = 5$ and $BC = 12$. Points M and N lie on $\overline{AC}$ and $\overline{BC}$, respectively, with $CM = CN = 4$. Points J and K are on $\overline{AB}$ so that $\overline{MJ}$ and $\overline{NK}$ are perpendicular to $\overline{AB}$. What is the area of pentagon $CMJKN$?



(A) 15 **(B)** $\frac{81}{5}$ **(C)** $\frac{205}{12}$ **(D)** $\frac{240}{13}$ **(E)** 20

2007B 14. Point P is inside equilateral $\triangle ABC$. Points Q, R, and S are the feet of the perpendiculars from P to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB?

(A) 4 **(B)** $3\sqrt{3}$ **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** 9

2010A 14. Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

(A) 30 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

2017B

15. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

(A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$