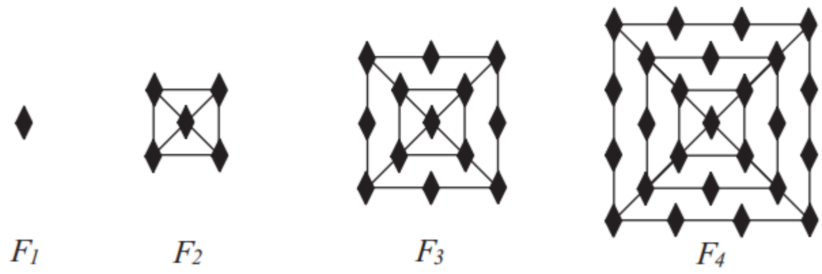
UNIT 21 EXERCISES 11-15

SERIES

2009A 11. The figures F_1, F_2, F_3 and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



(A) 401 **(B)** 485 **(C)** 585 **(D)** 626 **(E)** 761

2009B 11. On Monday, Millie puts a quart of seeds, 25% of which are millet, into a bird feeder. On each successive day she adds another quart of the same mix of seeds without removing any seeds that are left. Each day the birds eat only 25% of the millet in the feeder, but they eat all of the other seeds. On which day, just after Millie has placed the seeds, will the birds find that more than half the seeds in the feeder are millet?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday

(E) Saturday

2004B

12. In the sequence 2001, 2002, 2003, . . ., each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004^{th} term in this sequence?

(A) -2004 **(B)** -2 **(C)** 0 **(D)** 4003 **(E)** 6007

2008B

12. For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

(A) 2008 **(B)** 4015 **(C)** 4016 **(D)** 4,030,056 **(E)** 4,032,064

2009B

12. The fifth and eighth terms of a geometric sequence of real numbers are $7!$ and $8!$ respectively. What is the first term?

(A) 60 **(B)** 75 **(C)** 120 **(D)** 225 **(E)** 315

1999

13. Define a sequence of real numbers $a_1, a_2, a_3, \ldots$ by $a_1 = 1$ and $a_{n+1}^3 = 99a_n^3$ for all $n \geq 1$. Then a_{100} equals

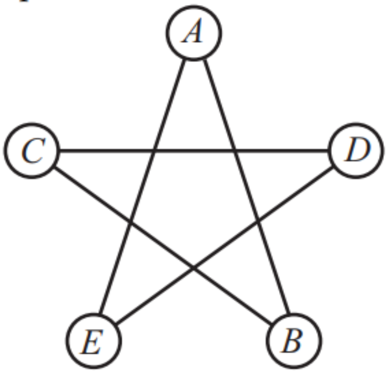
(A) 33^{33} **(B)** 33^{99} **(C)** 99^{33} **(D)** 99^{99} **(E)** none of these

2002B

13. The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

(A) 169 **(B)** 225 **(C)** 289 **(D)** 361 **(E)** 441

2005A 13. In the five-sided star shown, the letters A, B, C, D and E are replaced by the numbers 3, 5, 6, 7 and 9, although not necessarily in that order. The sums of the numbers at the ends of the line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$ form an arithmetic sequence, although not necessarily in that order. What is the middle term of the arithmetic sequence?



(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

2004A 14. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

(A) 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

2013A 14. The sequence

$$\log_{12} 162,\ \log_{12} x,\ \log_{12} y,\ \log_{12} z,\ \log_{12} 1250$$

is an arithmetic progression. What is x?

(A) $125\sqrt{3}$ **(B)** 270 **(C)** $162\sqrt{5}$ **(D)** 434 **(E)** $225\sqrt{6}$

2014A 14. Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value for c?

(A) -2 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

2016B 14. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

2007B

15. The geometric series $a + ar + ar^2 + \cdots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

(A) $\frac{4}{3}$ **(B)** $\frac{12}{7}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{7}{3}$ **(E)** $\frac{5}{2}$

2014B

15. When $p = \sum_{k=1}^{6} k \ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p?

(A) 2^{12} **(B)** 2^{14} **(C)** 2^{16} **(D)** 2^{18} **(E)** 2^{20}