UNIT 18 EXERCISES 11-15

ALGEBRA WORD PROBLEMS

2005A 11. How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

(A) 41 **(B)** 42 **(C)** 43 **(D)** 44 **(E)** 45

2007A 11. A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime number that always divides S?

(A) 3 **(B)** 7 **(C)** 13 **(D)** 37 **(E)** 43

2017B 11. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

(A) 1024 **(B)** 1524 **(C)** 1533 **(D)** 1536 **(E)** 2048

2003A 12. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2009A

12. How many positive integers less than 1000 are 6 times the sum of their digits?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 12

2011B

13. Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

2016B

12. All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What number is in the center?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2018A

12. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element of S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

2018A

13. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \leq i \leq 7$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** 59,048

2010B

14. Let a, b, c, d, and e be positive integers with $a + b + c + d + e = 2010$, and let M be the largest of the sums $a + b$, $b + c$, $c + d$ and $d + e$. What is the smallest possible value of M?

(A) 670 **(B)** 671 **(C)** 802 **(D)** 803 **(E)** 804

2012B

14. Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

2002B

15. How many four-digit numbers N have the property that the three-digit number obtained by removing the leftmost digit is one ninth of N?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2005B

15. The sum of four two-digit numbers is 221. None of the eight digits is 0 and no two of them are the same. Which of the following is **not** included among the eight digits?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2015A

15. What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123\,456\,789}{2^{26} \cdot 5^4}$ as a decimal?

(A) 4 **(B)** 22 **(C)** 26 **(D)** 30 **(E)** 104

2018B

15. How many 3-digit positive odd multiples of 3 do not include the digit 3 ?

(A) 96 **(B)** 97 **(C)** 98 **(D)** 102 **(E)** 120