UNIT 15 EXERCISES 11-15

ALGEBRA

1999

11. The student lockers at Olympic High are numbered consecutively beginning with locker number 1. The plastic digits used to number the lockers cost two cents apiece. Thus, it costs two cents to label locker number 9 and four cents to label locker number 10. If it costs \$137.94 to label all the lockers, how many lockers are there at the school?

(A) 2001 **(B)** 2010 **(C)** 2100 **(D)** 2726 **(E)** 6897

2007B

11. The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

(A) 125 **(B)** 144 **(C)** 153 **(D)** 173 **(E)** 180

2010A

11. The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

(A) $\frac{7}{15}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{8}{7}$ **(D)** $\frac{15}{8}$ **(E)** $\frac{15}{7}$

2012B

11. In the equation below, A and B are consecutive positive integers, and A, B, and $A + B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A + B$?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

1999

12. What is the maximum number of points of intersection of the graphs of two different fourth degree polynomial functions $y = p(x)$ and $y = q(x)$, each with leading coefficient 1?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

2001

12. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

(A) 768 **(B)** 801 **(C)** 934 **(D)** 1067 **(E)** 1167

2002B

12. For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 10

2010B 12. For what value of x does

$$\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 (x^2) + \log_8 (x^3) + \log_{16} (x^4) = 40\,?$$

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256 **(E)** 1024

2002A 13. Two different positive numbers a and b each differ from their reciprocals by 1. What is $a+b$?

(A) 1 **(B)** 2 **(C)** $\sqrt{5}$ **(D)** $\sqrt{6}$ **(E)** 3

2005B 13. Suppose that $4^{x_1} = 5$, $5^{x_2} = 6$, $6^{x_3} = 7, \ldots, 127^{x_{124}} = 128$. What is $x_1 x_2 \cdots x_{124}$?

(A) 2 **(B)** $\frac{5}{2}$ **(C)** 3 **(D)** $\frac{7}{2}$ **(E)** 4

2008B 14. A circle has a radius of $\log_{10}(a^2)$ and a circumference of $\log_{10}(b^4)$. What is $\log_a b$?

(A) $\frac{1}{4\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** π **(D)** 2π **(E)** $10^{2\pi}$

2018A 14. The solution to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\frac{1}{3}$ or $\frac{1}{2}$, can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

(A) 5 **(B)** 13 **(C)** 17 **(D)** 31 **(E)** 35

2008A 15. Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2009A

15. For what value of n is $i + 2i^2 + 3i^3 + \cdots + ni^n = 48 + 49i$?
Note: here $i = \sqrt{-1}$.

(A) 24 **(B)** 48 **(C)** 49 **(D)** 97 **(E)** 98

2009B

15. Assume $0 < r < 3$. Below are five equations for x. Which equation has the largest solution x?

(A) $3(1+r)^x = 7$ **(B)** $3(1+r/10)^x = 7$ **(C)** $3(1+2r)^x = 7$
(D) $3(1+\sqrt{r})^x = 7$ **(E)** $3(1+1/r)^x = 7$