UNIT 12 EXERCISES 11-15

PROBABILITY

2001 11. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{7}{10}$

2005B 11. An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is $20 or more?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

2010B 11. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

2012A 11. Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

(A) $\frac{5}{72}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** 1

2007A 12. Integers a, b, c, and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that $ad-bc$ is even?

(A) $\frac{3}{8}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{5}{8}$

2007B 13. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?

(A) $\frac{1}{63}$ **(B)** $\frac{1}{21}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{7}$ **(E)** $\frac{1}{3}$

2016A 13. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5)=1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N)<\frac{321}{400}$?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

2005A 14. On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

(A) $\frac{5}{11}$ **(B)** $\frac{10}{21}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{11}{21}$ **(E)** $\frac{6}{11}$

2011A 14. Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

(A) $\frac{11}{81}$ **(B)** $\frac{13}{81}$ **(C)** $\frac{5}{27}$ **(D)** $\frac{17}{81}$ **(E)** $\frac{19}{81}$

2010A 15. A coin is altered so that the probability that it lands on heads is less than than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

(A) $\frac{\sqrt{15}-3}{6}$ **(B)** $\frac{6-\sqrt{6\sqrt{6}+2}}{12}$ **(C)** $\frac{\sqrt{2}-1}{2}$ **(D)** $\frac{3-\sqrt{3}}{6}$

(E) $\frac{\sqrt{3}-1}{2}$

2012A

15. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

2015B

15. At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5 ?

(A) $\frac{11}{72}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{11}{24}$ **(E)** $\frac{1}{2}$