UNIT 19 EXERCISES 1-5

ALGEBRA WORD PROBLEMS

2000

1. In the year 2001, the United States will host the International Mathematical Olympiad. Let I, M, and O be distinct positive integers such that the product $I \cdot M \cdot O = 2001$. What is the largest possible value of the sum $I + M + O$?

(A) 23 **(B)** 55 **(C)** 99 **(D)** 111 **(E)** 671

2003A

1. What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 2003 **(E)** 4006

2001

2. Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

2004B 2. In the expression $c \cdot a^b - d$, the values of a, b, c, and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

2008B 2. A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

2016B 2. The harmonic mean of two numbers can be computed as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

(A) 2 **(B)** 45 **(C)** 504 **(D)** 1008 **(E)** 2015

2002B 3. For how many positive integers n is $n^2 - 3n + 2$ a prime number?

(A) none **(B)** one **(C)** two **(D)** more than two, but finitely many

(E) infinitely many

2003A 5. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

2004A 3. For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

(A) 33 **(B)** 49 **(C)** 50 **(D)** 99 **(E)** 100

2008B 3. A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least \$15,000, and that the total of all players' salaries for each team cannot exceed \$700,000. What is the maximum possible salary, in dollars, for a single player?

(A) 270,000 **(B)** 385,000 **(C)** 400,000 **(D)** 430,000 **(E)** 700,000

2014A 3. Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

1999 4. Find the sum of all prime numbers between 1 and 100 that are simultaneously 1 greater than a multiple of 4 and 1 less than a multiple of 5.

(A) 118 **(B)** 137 **(C)** 158 **(D)** 187 **(E)** 245

2005A 4. A store normally sells windows at \$100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How many dollars will they save if they purchase the windows together rather than separately?

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

2010A

4. If $x < 0$, then which of the following must be positive?

(A) $\frac{x}{|x|}$ **(B)** $-x^2$ **(C)** -2^x **(D)** $-x^{-1}$ **(E)** $\sqrt[3]{x}$

2001

5. What is the product of all positive odd integers less than 10,000?

(A) $\frac{10000!}{(5000!)^2}$ **(B)** $\frac{10000!}{2^{5000}}$ **(C)** $\frac{9999!}{2^{5000}}$ **(D)** $\frac{10000!}{2^{5000} \cdot 5000!}$ **(E)** $\frac{5000!}{2^{5000}}$

2007B

5. The 2007 AMC 12 contests will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

2010A

5. Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot a bullseye scores 10 points, with other possible scores being 8, 4, 2, and 0 points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bulleyes she will be guaranteed victory. What is the minimum value for n?

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

2010B

5. Lucky Larry's teacher asked him to substitute numbers for $a, b, c, d,$ and e in the expression $a - (b - (c - (d + e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for $a, b, c,$ and d were $1, 2, 3,$ and 4, respectively. What number did Larry substitute for e?

(A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5