UNIT 5 EXERCISES 6-10

CO-ORD GEO

2017B

6. **Answer (D):** The center of the circle is the midpoint of the diameter, which is $(4, 3)$, and the radius is $\sqrt{4^2 + 3^2} = 5$. Therefore the equation of the circle is $(x - 4)^2 + (y - 3)^2 = 25$. If $y = 0$, then $(x - 4)^2 = 16$, so $x = 0$ or $x = 8$. The circle intersects the x-axis at $(8, 0)$.

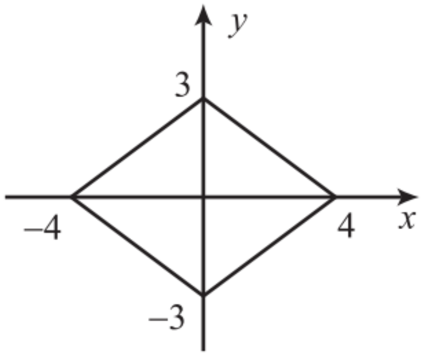
OR

Any diameter of a circle is a line of symmetry. Because the line $x = 4$ goes through the center of the circle, $(4, 3)$, it contains a diameter. The reflection of $(0, 0)$ in this line is $(8, 0)$. Alternatively, $(8, 6)$ can be reflected in the line $y = 3$, resulting in the same point.

2005B

7. **(D)** The graph is symmetric with respect to both coordinate axes, and in the first quadrant it coincides with the graph of the line $3x + 4y = 12$. Therefore the region is a rhombus, and the area is

$$\text{Area} = 4\left(\frac{1}{2}(4 \cdot 3)\right) = 24.$$



2016A

7. **Answer (D):** The given equation is equivalent to $(x^2 - y^2)(x + y + 1) = 0$, which is in turn equivalent to $(x + y)(x - y)(x + y + 1) = 0$. A product is 0 if and only if one of the factors is 0, so the graph is the union of the graphs of $x + y = 0$, $x - y = 0$, and $x + y + 1 = 0$. These are three straight lines, two of which intersect at the origin and the third of which does not pass through the origin. Therefore the graph consists of three lines that do not all pass through a common point.

2005B

8. **(C)** The vertex of the parabola is $(0, a^2)$. The line passes through the vertex if and only if $a^2 = 0 + a$. There are two solutions to this equation, namely $a = 0$ and $a = 1$.

2006B 8. **(E)** Substituting $x = 1$ and $y = 2$ into the equations gives

$$1 = \frac{2}{4} + a \quad \text{and} \quad 2 = \frac{1}{4} + b.$$

It follows that

$$a + b = \left(1 - \frac{2}{4}\right) + \left(2 - \frac{1}{4}\right) = 3 - \frac{3}{4} = \frac{9}{4}.$$

OR

Because

$$a = x - \frac{y}{4} \quad \text{and} \quad b = y - \frac{x}{4}, \quad \text{we have} \quad a + b = \frac{3}{4}(x + y).$$

Since $x = 1$ when $y = 2$, this implies that $a + b = \frac{3}{4}(1 + 2) = \frac{9}{4}$.

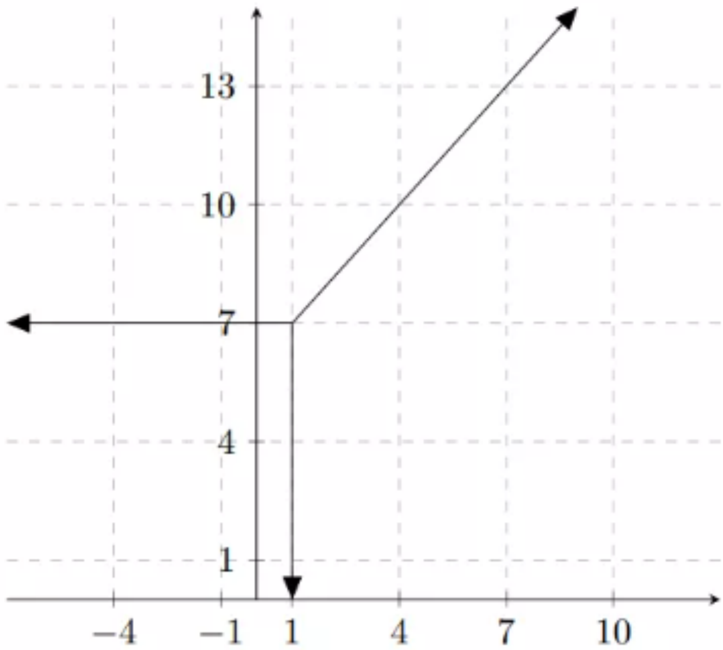
2014A 8. **Answer (C):** Let $P > 100$ be the listed price. Then the price reductions in dollars are as follows:

Coupon 1: $\frac{P}{10}$
Coupon 2: 20
Coupon 3: $\frac{18}{100}(P - 100)$

Coupon 1 gives a greater price reduction than coupon 2 when $\frac{P}{10} > 20$, that is, $P > 200$. Coupon 1 gives a greater price reduction than coupon 3 when $\frac{P}{10} > \frac{18}{100}(P - 100)$, that is, $P < 225$. The only choice that satisfies these inequalities is \$219.95.

2017A

9. **Answer (E):** Suppose that the two larger quantities are the first and the second. Then $3 = x + 2 \geq y - 4$. This is equivalent to $x = 1$ and $y \leq 7$, and its graph is the downward-pointing ray with endpoint $(1, 7)$. Similarly, if the two larger quantities are the first and third, then $3 = y - 4 \geq x + 2$. This is equivalent to $y = 7$ and $x \leq 1$, and its graph is the leftward-pointing ray with endpoint $(1, 7)$. Finally, if the two larger quantities are the second and third, then $x + 2 = y - 4 \geq 3$. This is equivalent to $y = x + 6$ and $x \geq 1$, and its graph is the ray with endpoint $(1, 7)$ that points upward and to the right. Thus the graph consists of three rays with common endpoint $(1, 7)$.



Note: This problem is related to a relatively new area of mathematics called tropical geometry.

2017B

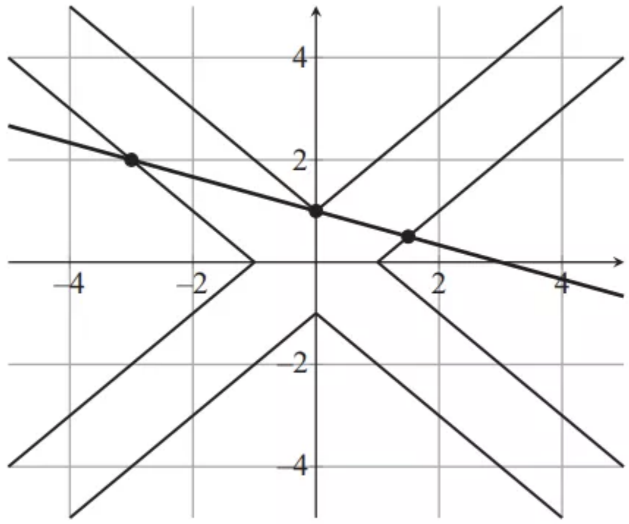
9. **Answer (A):** The first circle has equation $(x+10)^2+(y+4)^2 = 169$, and the second circle has equation $(x-3)^2+(y-9)^2 = 65$. Expanding these two equations, subtracting, and simplifying yields $x + y = 3$. Because the points of intersection of the two circles must satisfy this new equation, it must be the required equation of the line through those points, so $c = 3$. In fact, the circles intersect at $(2, 1)$ and $(-5, 8)$.

1999

10. **(C)** Since both I and III cannot be false, the digit must be 1 or 3. So either I or III is the false statement. Thus II and IV must be true and (C) is necessarily correct. For the same reason, (E) must be incorrect. If the digit is 1, (B) and (D) are incorrect, and if the digit is 3, (A) is incorrect.

2018A

10. **Answer (C):** The graph of the system is shown below.



The graph of the first equation is a line with x-intercept $(3, 0)$ and y-intercept $(0, 1)$. To draw the graph of the second equation, consider the equation quadrant by quadrant. In the first quadrant $x > 0$ and $y > 0$, and thus the second equation is equivalent to $|x-y| = 1$, which in turn is equivalent to $y = x \pm 1$. Its graph consists of the rays with endpoints $(0, 1)$ and $(1, 0)$, as shown. In the second quadrant $x < 0$ and $y > 0$. The corresponding graph is the reflection of the first quadrant graph across the y-axis. The rest of the graph can be sketched by further reflections of the first-quadrant graph across the coordinate axes, resulting in the figure shown. There are 3 intersection points: $(-3, 2)$, $(0, 1)$, and $\left(\frac{3}{2}, \frac{1}{2}\right)$, as shown.

OR

The second equation implies that $x = y \pm 1$ or $x = -y \pm 1$. There are four cases:

- If $x = y + 1$, then $(y + 1) + 3y = 3$, so $(x, y) = \left(\frac{3}{2}, \frac{1}{2}\right)$.
- If $x = y - 1$, then $(y - 1) + 3y = 3$, so $(x, y) = (0, 1)$.
- If $x = -y + 1$, then $(-y + 1) + 3y = 3$, so again $(x, y) = (0, 1)$.
- If $x = -y - 1$, then $(-y - 1) + 3y = 3$, so $(x, y) = (-3, 2)$.

It may be checked that each of these ordered pairs actually satisfies the given equations, so the total number of solutions is 3.