UNIT 4 EXERCISES 6-10

TRIANGLES

2004B 6. **(A)** Let downtown St. Paul, downtown Minneapolis, and the airport be located at S, M, and A, respectively. Then $\triangle MAS$ has a right angle at A, so by the Pythagorean Theorem,

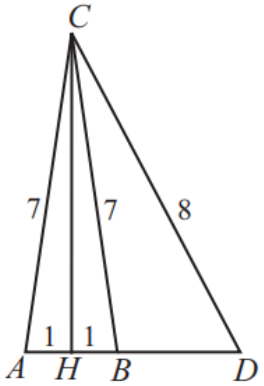
$$MS = \sqrt{10^2 + 8^2} = \sqrt{164} \approx \sqrt{169} = 13.$$

2005B

6. **(A)** Let $\overline{CH}$ be an altitude of $\triangle ABC$. Applying the Pythagorean Theorem to $\triangle CHB$ and to $\triangle CHD$ produces

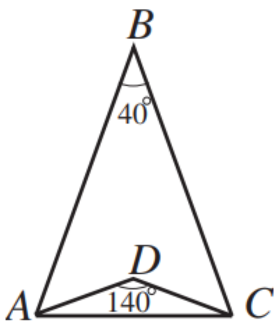
$$8^2 - (BD+1)^2 = CH^2 = 7^2 - 1^2 = 48, \quad \text{so} \quad (BD+1)^2 = 16.$$

Thus $BD = 3$.



2007A

6. **Answer (D):** Because $\triangle ABC$ is isosceles, $\angle BAC = \frac{1}{2}(180^\circ - \angle ABC) = 70^\circ$.



Similarly,

$$\angle DAC = \frac{1}{2}(180^\circ - \angle ADC) = 20^\circ.$$

Thus $\angle BAD = \angle BAC - \angle DAC = 50^\circ$.

OR

Because $\triangle ABC$ and $\triangle ADC$ are isosceles triangles, applying the Exterior Angle Theorem to $\triangle ABD$ gives $\angle BAD = 70^\circ - 20^\circ = 50^\circ$.

2007B

6. **Answer (D):** The perimeter of the triangle is $5+6+7=18$, so the distance that each bug crawls is 9. Therefore $AB+BD=9$, and $BD=4$.

2016B

6. **Answer (C):** Let the vertex of the triangle that lies in the first quadrant be (x, x^2). Then the base of the triangle is $2x$ and the height is x^2, so $\frac{1}{2}\cdot 2x \cdot x^2 = 64$. Thus $x^3 = 64$, $x = 4$, and $BC = 2x = 8$.

2003A

7. (B) The longest side cannot be greater than 3, since otherwise the remaining two sides would not be long enough to form a triangle. The only possible triangles have side lengths 1–3–3 or 2–2–3.

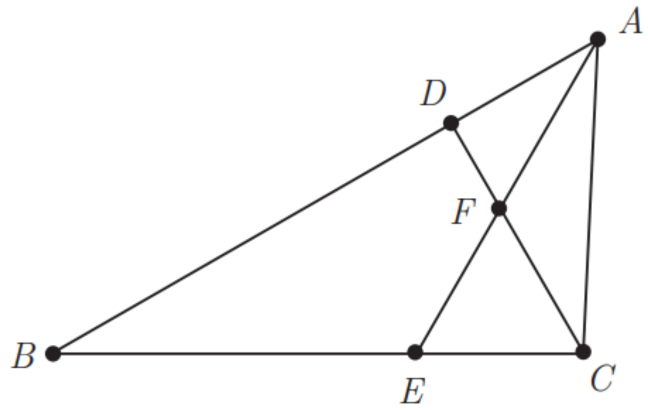
2010A 8. **Answer (C):** Let $\alpha = \angle BAE = \angle ACD = \angle ACF$. Because $\triangle CFE$ is equilateral, it follows that $\angle CFA = 120°$ and then

$$\angle FAC = 180° - 120° - \angle ACF = 60° - \alpha.$$

Therefore

$$\angle BAC = \angle BAE + \angle FAC = \alpha + (60° - \alpha) = 60°.$$

Because $AB = 2 \cdot AC$, it follows that $\triangle BAC$ is a $30-60-90°$ triangle, and thus $\angle ACB = 90°$.



2009B 9. **Answer (A):** Because the line $x + y = 7$ is parallel to $\overline{AB}$, the area of $\triangle ABC$ is independent of the location of C on the line. Therefore it may be assumed that $C = (7, 0)$. In that case the triangle has base $AC = 4$ and altitude 3, so its area is $\frac{1}{2} \cdot 4 \cdot 3 = 6$.

OR

The base of the triangle is $AB = \sqrt{3^2 + 3^2} = 3\sqrt{2}$. Its altitude is the distance between the point A and the parallel line $x + y = 7$, which is

$$\frac{|3 + 0 - 7|}{\sqrt{1^2 + 1^2}} = 2\sqrt{2}.$$

Therefore its area is $\frac{1}{2} \cdot 3\sqrt{2} \cdot 2\sqrt{2} = 6$.

2006B

10. **(A)** The sides of the triangle are x, $3x$, and 15 for some positive integer x. By the Triangle Inequality, these three numbers are the sides of a triangle if and only if $x+3x > 15$ and $x+15 > 3x$. Because x is an integer, the first inequality is equivalent to $x \geq 4$, and the second inequality is equivalent to $x \leq 7$. Thus the greatest possible perimeter is $7+21+15 = 43$.

2007A

10. **Answer (A):** Let the sides of the triangle have lengths $3x$, $4x$, and $5x$. The triangle is a right triangle, so its hypotenuse is a diameter of the circle. Thus $5x = 2 \cdot 3 = 6$, so $x = 6/5$. The area of the triangle is

$$\frac{1}{2} \cdot 3x \cdot 4x = \frac{1}{2} \cdot \frac{18}{5} \cdot \frac{24}{5} = \frac{216}{25} = 8.64.$$

OR

A right triangle with side lengths 3, 4, and 5 has area $(1/2)(3)(4) = 6$. Because the given right triangle is inscribed in a circle with diameter 6, the hypotenuse of this triangle has length 6. Thus the sides of the given triangle are $6/5$ as long as those of a $3-4-5$ triangle, and its area is $(6/5)^2$ times that of a $3-4-5$ triangle. The area of the given triangle is

$$\left(\frac{6}{5}\right)^2 (6) = \frac{216}{25} = 8.64.$$

2015B

10. **Answer (C):** Let the side lengths be $a < b < c$. By the Triangle Inequality $a + b > c$; it follows that perimeter $P = a + b + c > 2c$. Then $2c < P < 15$, $2c < 14$, and $c < 7$. The only triangles (denoted by three-digit numbers with decreasing digits) that are not equilateral or isosceles are 653, 652, 643, 543, 542, and 432. Of these, only 543 is a right triangle, so the answer is 5.