UNIT 3 EXERCISES 6-10

2D GEO WORD

2011B

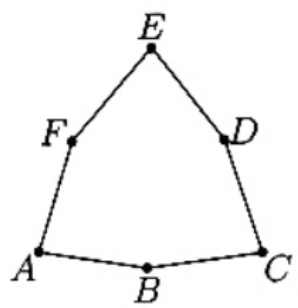
6. **Answer (C):** Let O be the center of the circle, and let the degree measures of the minor and major arcs be $2x$ and $3x$, respectively. Because $2x + 3x = 360°$, it follows that $x = 72°$ and $\angle BOC = 2x = 144°$. In quadrilateral $ABOC$, the segments AB and AC are tangent to the circle, thus $\angle ABO = \angle ACO = 90°$ and $\angle BAC = 360° - (144° + 90° + 90°) = 36°$.

2017A

6. **Answer (B):** Four rods can form a quadrilateral with positive area if and only if the length of the longest rod is less than the sum of the lengths of the other three. Therefore if the fourth rod has length n cm, then n must satisfy the inequalities $15 < 3+7+n$ and $n < 3+7+15$, that is, $5 < n < 25$. Because n is an integer, it must be one of the 19 integers from 6 to 24, inclusive. However, the rods of lengths 7 cm and 15 cm have already been chosen, so the number of rods that Joy can choose is $19 - 2 = 17$.

1999

7. **(B)** The sum of the angles in a convex hexagon is 720° and each angle must be less than 180°. If four of the angles are acute, then their sum would be less than 360°, and therefore at least one of the two remaining angles would be greater than 180°, a contradiction. Thus there can be at most three acute angles. The hexagon shown has three acute angles, A, C, and E.



OR

The result holds for *any* convex n-gon. The sum of the exterior angles of a convex n-gon is 360°. Hence at most three of these angles can be obtuse, for otherwise the sum would exceed 360°. Thus the largest number of acute angles in any convex n-gon is three.

2002A

7. **(A)** Let $C_A = 2\pi R_A$ be the circumference of circle A, let $C_B = 2\pi R_B$ be the circumference of circle B, and let L the common length of the two arcs. Then

$$\frac{45}{360}C_A = L = \frac{30}{360}C_B.$$

Therefore

$$\frac{C_A}{C_B} = \frac{2}{3} \quad \text{so} \quad \frac{2}{3} = \frac{2\pi R_A}{2\pi R_B} = \frac{R_A}{R_B}.$$

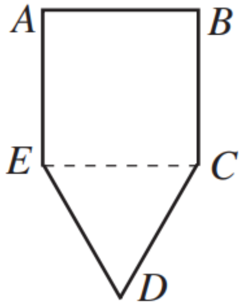
Thus, the ratio of the areas is

$$\frac{\text{Area of Circle }(A)}{\text{Area of Circle }(B)} = \frac{\pi R_A^2}{\pi R_B^2} = \left(\frac{R_A}{R_B}\right)^2 = \frac{4}{9}.$$

2004B 7. **(B)** The areas of the regions enclosed by the square and the circle are $10^2 = 100$ and $\pi(10)^2 = 100\pi$, respectively. One quarter of the second region is also included in the first, so the area of the union is

$$100 + 100\pi - 25\pi = 100 + 75\pi.$$

2007B 7. **Answer (E):** Because $AB = BC = EA$ and $\angle A = \angle B = 90°$, quadrilateral $ABCE$ is a square, so $\angle AEC = 90°$.



Also $CD = DE = EC$, so $\triangle CDE$ is equilateral and $\angle CED = 60°$. Therefore

$$\angle E = \angle AEC + \angle CED = 90° + 60° = 150°.$$

2015B 7. **Answer (D):** The lines of symmetry are the 15 lines joining a vertex to the midpoint of the opposite side, so $L = 15$. There is rotational symmetry around the center of the 15-gon, and the smallest positive angle of rotation that will transform the 15-gon onto itself is $\frac{360}{15} = 24$ degrees; therefore $R = 24$. The sum is $15 + 24 = 39$.

2007A 8. **Answer (C):** Consider the two chords with an endpoint at 5. The arc subtended by the angle determined by these chords extends from 10 to 12, so the degree measure of the arc is $(2/12)(360) = 60$. By the Central Angle Theorem, the degree measure of this angle is $(1/2)(60) = 30$. By symmetry, the degree measure of the angle at each vertex is 30.

2008B 8. **Answer (C):** Because $AB + BD = AD$ and $AB = 4BD$, it follows that $BD = \frac{1}{5} \cdot AD$. By similar reasoning, $CD = \frac{1}{10} \cdot AD$. Thus

$$BC = BD - CD = \frac{1}{5} \cdot AD - \frac{1}{10} \cdot AD = \frac{1}{10} \cdot AD.$$

2015A 8. **Answer (C):** Let the sides of the rectangle have lengths $3a$ and $4a$. By the Pythagorean Theorem, the diagonal has length $5a$. Because $5a = d$, the side lengths are $\frac{3}{5}d$ and $\frac{4}{5}d$. Therefore the area is $\frac{3}{5}d \cdot \frac{4}{5}d = \frac{12}{25}d^2$, so $k = \frac{12}{25}$.

2017B

8. **Answer (C):** Let x be the length of the short side of the rectangle, and let y be the length of the long side. Then the length of the diagonal is $\sqrt{x^2+y^2}$, and

$$\frac{x^2}{y^2}=\frac{y^2}{x^2+y^2}, \qquad \text{so} \qquad \frac{y^2}{x^2}=\frac{x^2+y^2}{y^2}=\frac{x^2}{y^2}+1.$$

Let $r=\frac{x^2}{y^2}$ be the requested squared ratio. Then $\frac{1}{r}=r+1$, so $r^2+r-1=0$. By the quadratic formula, the positive solution is $r=\frac{\sqrt{5}-1}{2}$.

2018B

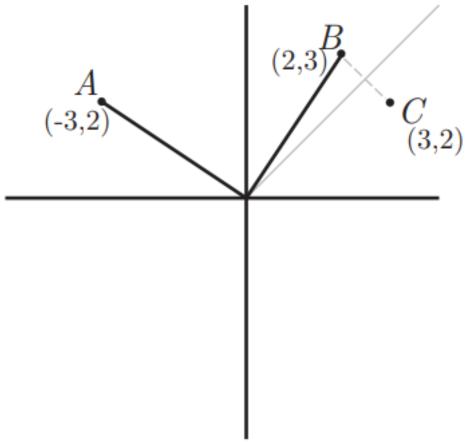
8. **Answer (C):** Let O be the center of the circle. Triangle ABC is a right triangle, and O is the midpoint of the hypotenuse $\overline{AB}$. Then $\overline{OC}$ is a radius, and it is also one of the medians of the triangle. The centroid is located one third of the way along the median from O to C, so the centroid traces out a circle with center O and radius $\frac{1}{3}\cdot 12=4$ (except for the two missing points corresponding to $C=A$ or $C=B$). The area of this smaller circle is then $\pi\cdot 4^2=16\pi\approx 16\cdot 3.14\approx 50$.

2003A

9. (D) The set S is symmetric about the line $y=x$ and contains $(2,3)$, so it must also contain $(3,2)$. Also S is symmetric about the x-axis, so it must contain $(2,-3)$ and $(3,-2)$. Finally, since S is symmetric about the y-axis, it must contain $(-2,3)$, $(-3,2)$, $(-2,-3)$, and $(-3,-2)$. Since the resulting set of 8 points is symmetric about both coordinate axes, it is also symmetric about the origin.

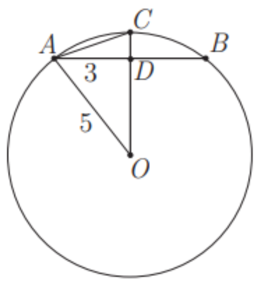
2004B

9. **(E)** The rotation takes $(-3, 2)$ into $B = (2, 3)$, and the reflection takes B into $C = (3, 2)$.



2008B

9. **Answer (A):** Let O be the center of the circle, and let D be the intersection of $\overline{OC}$ and $\overline{AB}$. Because $\overline{OC}$ bisects minor arc AB, $\overline{OD}$ is a perpendicular bisector of chord $\overline{AB}$. Hence $AD = 3$, and applying the Pythagorean Theorem to $\triangle ADO$ yields $OD = \sqrt{5^2 - 3^3} = 4$. Therefore $DC = 1$, and applying the Pythagorean Theorem to $\triangle ADC$ yields $AC = \sqrt{3^2 + 1^2} = \sqrt{10}$.

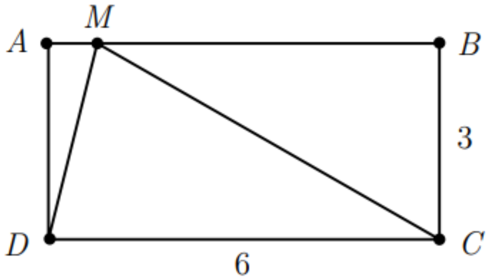


2016B

9. **Answer (B):** Let x be the number of posts along the shorter side; then there are $2x$ posts along the longer side. When counting the number of posts on all the sides of the garden, each corner post is counted twice, so $2x + 2(2x) = 20 + 4$. Solving this equation gives $x = 4$. Thus the dimensions of the rectangle are $(4 - 1) \cdot 4 = 12$ yards by $(8 - 1) \cdot 4 = 28$ yards. The requested area is given by the product of these dimensions, $12 \cdot 28 = 336$ square yards.

2011B

10. **Answer (E):** Sides $\overline{AB}$ and $\overline{CD}$ are parallel, so $\angle CDM = \angle AMD$. Because $\angle AMD = \angle CMD$, it follows that $\triangle CMD$ is isosceles and $CD = CM = 6$. Therefore $\triangle MCB$ is a $30-60-90^\circ$ right triangle with $\angle BMC = 30^\circ$. Finally, $2 \cdot \angle AMD + 30^\circ = \angle AMD + \angle CMD + 30^\circ = 180^\circ$, so $\angle AMD = 75^\circ$.



2016B

10. **Answer (A):** The slopes of $\overline{PQ}$ and $\overline{RS}$ are -1, and the slopes of $\overline{QR}$ and $\overline{PS}$ are 1, so the figure is a rectangle. The side lengths are $PQ = (a-b)\sqrt{2}$ and $PS = (a+b)\sqrt{2}$, so the area is $2(a-b)(a+b) = 2(a^2-b^2) = 16$. Therefore $a^2 - b^2 = 8$. The only perfect squares whose difference is 8 are 9 and 1, so $a = 3$, $b = 1$, and $a + b = 4$.