UNIT 23 EXERCISES 6-10

FUNCTIONS

2003A

6. (C) For example, $-1\heartsuit 0 = |-1-0| = 1 \neq -1$. All the other statements are true:
(A) $x\heartsuit y = |x-y| = |-(y-x)| = |y-x| = y\heartsuit x$ for all x and y.
(B) $2(x\heartsuit y) = 2|x-y| = |2x-2y| = (2x)\heartsuit(2y)$ for all x and y.
(D) $x\heartsuit x = |x-x| = 0$ for all x.
(E) $x\heartsuit y = |x-y| > 0$ if $x \neq y$.

2017A

7. **Answer (B):** It is clear after listing the first few values, $f(1) = 2$, $f(2) = f(1) + 1 = 3$, $f(3) = f(1) + 2 = 4$, $f(4) = f(3) + 1 = 5$, and so on, that $f(n) = n + 1$ for all positive integers n. Indeed, the function is uniquely determined by the recursive description, and the function defined by $f(n) = n + 1$ fits the description. Therefore $f(2017) = 2018$.

2003B

8. (E) Let $y = \clubsuit(x)$. Since $x \leq 99$, we have $y \leq 18$. Thus if $\clubsuit(y) = 3$, then $y = 3$ or $y = 12$. The 3 values of x for which $\clubsuit(x) = 3$ are 12, 21, and 30, and the 7 values of x for which $\clubsuit(x) = 12$ are 39, 48, 57, 66, 75, 84, and 93. There are 10 values in all.

2001

9. **(C)** Note that

$$f(600) = f(500 \cdot \frac{6}{5}) = \frac{f(500)}{6/5} = \frac{3}{6/5} = \frac{5}{2}.$$

OR

For all positive x,

$$f(x) = f(1 \cdot x) = \frac{f(1)}{x},$$

so $xf(x)$ is the constant $f(1)$. Therefore,

$$600f(600) = 500f(500) = 500(3) = 1500,$$

so $f(600) = \frac{1500}{600} = \frac{5}{2}$. Note. $f(x) = \frac{1500}{x}$ is the unique function satisfying the given conditions.

2003B 9. (D) Since f is a linear function, its slope is constant. Therefore

$$\frac{f(6)-f(2)}{6-2}=\frac{f(12)-f(2)}{12-2}, \quad \text{so} \quad \frac{12}{4}=\frac{f(12)-f(2)}{10},$$

and $f(12)-f(2)=30$.

OR

Since f is a linear function, it has a constant rate of change, given by

$$\frac{f(6)-f(2)}{6-2}=\frac{12}{4}=3.$$

Therefore $f(12)-f(2)=3(12-2)=30$.

OR

If $f(x)=mx+b$, then

$$12=f(6)-f(2)=6m+b-(2m+b)=4m,$$

so $m=3$. Hence

$$f(12)-f(2)=12m+b-(2m+b)=10m=30.$$