UNIT 22 EXERCISES 6-10

QUADRATICS

2002B 6. **(C)** The given conditions imply that

$$x^2 + ax + b = (x - a)(x - b) = x^2 - (a + b)x + ab,$$

so

$$a + b = -a \quad \text{and} \quad ab = b.$$

Since $b \neq 0$, the second equation implies that $a = 1$. The first equation gives $b = -2$, so $(a, b) = (1, -2)$.

2014A 6. **Answer (D):** Let $10a + b$ be the larger number. Then $10a + b - (10b + a) = 5(a + b)$, which simplifies to $2a = 7b$. The only nonzero digits that satisfy this equation are $a = 7$ and $b = 2$. Therefore the larger number is 72, and the required sum is $72 + 27 = 99$.

2005A 9. **(A)** The quadratic formula yields

$$x = \frac{-(a+8) \pm \sqrt{(a+8)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}.$$

The equation has only one solution precisely when the value of the discriminant, $(a + 8)^2 - 144$, is 0. This implies that $a = -20$ or $a = 4$, and the sum is -16.

OR

The equation has one solution if and only if the polynomial is the square of a binomial with linear term $\pm\sqrt{4x^2} = \pm 2x$ and constant term $\pm\sqrt{9} = \pm 3$. Because $(2x \pm 3)^2$ has a linear term $\pm 12x$, it follows that $a + 8 = \pm 12$. Thus a is either -20 or 4, and the sum of those values is -16.

2009A 9. **Answer (D):** Expanding, we have $f(x+3) = a(x^2+6x+9)+b(x+3)+c = ax^2+(6a+b)x+(9a+3b+c)$. Equating coefficients implies that $a = 3$, $6\cdot3+b = 7$, whence $b = -11$, and then $9\cdot3+3\cdot(-11)+c = 4$, and so $c = 10$. Therefore $a+b+c = 3-11+10 = 2$.

OR

Note that

$$\begin{aligned} f(x) = f((x-3)+3) &= 3(x-3)^2+7(x-3)+4 \\ &= 3(x^2-6x+9)+7x-21+4 \\ &= 3x^2-11x+10. \end{aligned}$$

Therefore $a = 3$, $b = -11$, and $c = 10$, giving $a+b+c = 2$.

OR

The sum $a+b+c$ is $f(1) = f(-2+3) = 3(-2)^2+7(-2)+4 = 2$.

2007B 9. **Answer (A):** Let $u = 3x-1$. Then $x = (u+1)/3$, and

$$f(u) = \left(\frac{u+1}{3}\right)^2 + \frac{u+1}{3} + 1 = \frac{u^2+2u+1}{9} + \frac{u+1}{3} + 1 = \frac{u^2+5u+13}{9}.$$

In particular,

$$f(5) = \frac{5^2+5\cdot5+13}{9} = \frac{63}{9} = 7.$$

OR

The value of $3x-1$ is 5 when $x = 2$. Thus

$$f(5) = f(3\cdot2-1) = 2^2+2+1 = 7.$$

2012B

10. **Answer (B):** Solve the first equation for y^2 and substitute into the second equation to get $x^2+x-20=0$, so $x=4$ or $x=-5$. This leads to the intersection points $(-5,0)$, $(4,3)$, and $(4,-3)$. The vertical side of the triangle with these three vertices has length $3-(-3)=6$, and the horizontal height to that side has length $4-(-5)=9$, so its area is $\frac{1}{2}\cdot 6\cdot 9=27$.