UNIT 17 EXERCISES 6-10

ARITHMETIC

1999

6. **(D)** Note that

$$2^{1999} \cdot 5^{2001} = 2^{1999} \cdot 5^{1999} \cdot 5^2 = 10^{1999} \cdot 25 = 25\overbrace{0\ldots0}^{1999\text{ zeros}}.$$

Hence the sum of the digits is 7.

2004A

6. **(A)** Since none of V, W, X, Y, or Z exceeds 2004^{2005}, the difference $U - V = 2004^{2005}$ is the largest.

2009B

6. **Answer (C):** The three operations can be performed in any of $3! = 6$ orders. However, if the addition is performed either first or last, then multiplying in either order produces the same result. Thus at most four distinct values can be obtained. It is easily checked that the values of the four expressions

$$(2 \times 3) + (4 \times 5) = 26,$$
$$((2 \times 3 + 4) \times 5) = 50,$$
$$2 \times (3 + (4 \times 5)) = 46,$$
$$2 \times (3 + 4) \times 5 = 70$$

are in fact all distinct.

2018B 7. **Answer (C):** The change of base formula states that $\log_a b = \frac{\log b}{\log a}$. Thus the product telescopes:

$$\begin{aligned}\frac{\log 7}{\log 3}\cdot\frac{\log 9}{\log 5}\cdot\frac{\log 11}{\log 7}\cdot\frac{\log 13}{\log 9}\cdots\frac{\log 25}{\log 21}\cdot\frac{\log 27}{\log 23} &= \frac{\log 25}{\log 3}\cdot\frac{\log 27}{\log 5}\\ &= \frac{\log 5^2}{\log 3}\cdot\frac{\log 3^3}{\log 5}\\ &= \frac{2\log 5}{\log 3}\cdot\frac{3\log 3}{\log 5}.\\ &= 6.\end{aligned}$$

OR

Let

$$a = \log_3 7\cdot\log_7 11\cdot\log_{11} 15\cdot\log_{15} 19\cdot\log_{19} 23\cdot\log_{23} 27$$

and

$$b = \log_5 9\cdot\log_9 13\cdot\log_{13} 17\cdot\log_{17} 21\cdot\log_{21} 25.$$

The required product is ab. Now

$$\begin{aligned}b &= \log_5 9\cdot\log_9 13\cdot\log_{13} 17\cdot\log_{17} 21\cdot\log_{21} 25\\ &= \log_5 9^{\log_9 13}\cdot\log_{13} 17\cdot\log_{17} 21\cdot\log_{21} 25\\ &= \log_5 13\cdot\log_{13} 17\cdot\log_{17} 21\cdot\log_{21} 25\\ &= \log_5 13^{\log_{13} 17}\cdot\log_{17} 21\cdot\log_{21} 25\\ &= \log_5 17\cdot\log_{17} 21\cdot\log_{21} 25\\ &= \log_5 17^{\log_{17} 21}\cdot\log_{21} 25\\ &= \log_5 21\cdot\log_{21} 25\\ &= \log_5 21^{\log_{21} 25}\\ &= \log_5 25\\ &= 2.\end{aligned}$$

Similarly, $a = \log_3 27 = 3$, so $ab = 2\cdot 3 = 6$.

2015B **8. Answer (D):**

$$\left(625^{\log_5 2015}\right)^{\frac{1}{4}} = \left((5^4)^{\log_5 2015}\right)^{\frac{1}{4}} = \left(5^{4\log_5 2015}\right)^{\frac{1}{4}} = \left(5^{\log_5 2015}\right)^{4\cdot\frac{1}{4}} = 2015$$