UNIT 12 EXERCISES 6-10

PROBABILITY

2003A 8. (E) The factors of 60 are

$$1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, \text{ and } 60.$$

Six of the twelve factors are less than 7, so the probability is $1/2$.

2015B 9. **Answer (C):** Let x be the probability that Larry wins the game. Then $x = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot x$. To see this, note that Larry can win by knocking the bottle off the ledge on his first throw; if he and Julius both miss, then it is as if they started the game all over. Thus $x = \frac{1}{2} + \frac{1}{4}x$, so $\frac{3}{4}x = \frac{1}{2}$ or $x = \frac{2}{3}$.

OR

For Larry to win on his nth throw, there must be $2n-2$ misses—$n-1$ by Larry and $n-1$ by Julius—followed by a hit by Larry. Because the probability of each of these independent events is $\frac{1}{2}$, the probability that Larry wins on his nth throw is $\left(\frac{1}{2}\right)^{2n-1}$. Therefore the probability that Larry wins the game is given by a geometric series:

$$\begin{aligned}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{2n-1} &= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots \\ &= \frac{1}{2}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \cdots\right) \\ &= \frac{1}{2} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}.\end{aligned}$$

2011B 9. **Answer (D):** Consider all ordered pairs (a, b) with each of the numbers a and b in the closed interval $[-20, 10]$. These pairs fill a 30×30 square in the coordinate plane, with an area of 900 square units. Ordered pairs in the first and third quadrants have the desired property, namely $a \cdot b > 0$. The areas of the portions of the 30×30 square in the first and third quadrants are $10^2 = 100$ and $20^2 = 400$, respectively. Therefore the probability of a positive product is $\frac{100+400}{900} = \frac{5}{9}$.

OR

Each of the numbers is positive with probability $\frac{1}{3}$ and negative with probability $\frac{2}{3}$. Their product is positive if and only if both numbers are positive or both are negative, so the requested probability is $\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{5}{9}$.

2015A

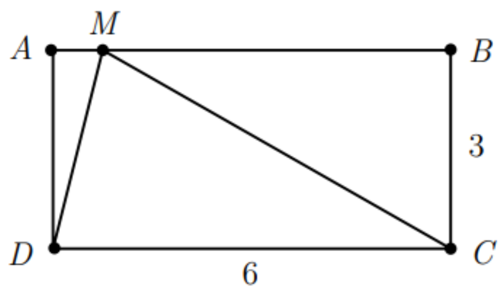
9. **Answer (C):** Because the marbles left for Cheryl are determined at random, the second of Cheryl's marbles is equally likely to be any of the 5 marbles other than her first marble. One of those 5 marbles matches her first marble in color. Therefore the probability is $\frac{1}{5}$.

OR

Because all the choices are made at random, Cheryl is equally likely to take any of the $\binom{6}{2} = 15$ possible pairs of marbles. Exactly 3 of these are pairs of same-colored marbles. Therefore the requested probability is $\frac{3}{15} = \frac{1}{5}$.

2011A

10. **Answer (E):** Sides $\overline{AB}$ and $\overline{CD}$ are parallel, so $\angle CDM = \angle AMD$. Because $\angle AMD = \angle CMD$, it follows that $\triangle CMD$ is isosceles and $CD = CM = 6$. Therefore $\triangle MCB$ is a $30-60-90°$ right triangle with $\angle BMC = 30°$. Finally, $2 \cdot \angle AMD + 30° = \angle AMD + \angle CMD + 30° = 180°$, so $\angle AMD = 75°$.



2017A 10. **Answer (C):** Half of the time Laurent will pick a number between 2017 and 4034, in which case the probability that his number will be greater than Chloé's number is 1. The other half of the time, he will pick a number between 0 and 2017, and by symmetry his number will be the larger one in half of those cases. Therefore the requested probability is $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$.

OR

The choices of numbers can be represented in the coordinate plane by points in the rectangle with vertices at $(0, 0)$, $(2017, 0)$, $(2017, 4034)$, and $(0, 4034)$. The portion of the rectangle representing the event that Laurent's number is greater than Chloé's number is the portion above the line segment with endpoints $(0, 0)$ and $(2017, 2017)$. This area is $\frac{3}{4}$ of the area of the entire rectangle, so the requested probability is $\frac{3}{4}$.