UNIT 11 EXERCISES 6-10

STATS

2018A 6. **Answer (B):** Note that the given conditions imply that the 6 values are listed in increasing order. Because the median of the these 6 values is n, the mean of the middle two values must be n, so

$$\frac{(m+10)+(n+1)}{2}=n,$$

which implies $m=n-11$. Because the mean of the set is also n,

$$\frac{(n-11)+(n-7)+(n-1)+(n+1)+(n+2)+2n}{6}=n,$$

so $7n-16=6n$ and $n=16$. Then $m=16-11=5$, and the requested sum is $5+16=21$.

2011B 7. **Answer (B):** Because $x \le 99$ and $\frac{1}{2}(x+y)=60$, it follows that $y=120-x \ge 120-99=21$. Thus the maximum value of $\frac{x}{y}$ is $\frac{99}{21}=\frac{33}{7}$.

2010B 8. **Answer (B):** If there are n schools in the city, then there are $3n$ contestants, so $3n \geq 64$, and $n \geq 22$. Because Andrea received the median score and each student received a different score, n is odd, so $n \geq 23$. Andrea's position is $\frac{3n+1}{2}$, and Andrea finished ahead of Beth, so $\frac{3n+1}{2} < 37$, and $3n < 73$. Because n is an odd integer, $n \leq 23$. Therefore $n = 23$.

2012A 8. **Answer (C):** If the numbers are arranged in the order a, b, c, d, e, then the iterative average is

$$\frac{\frac{\frac{\frac{a+b}{2}+c}{2}+d}{2}+e}{2} = \frac{a+b+2c+4d+8e}{16}.$$

The largest value is obtained by letting $(a, b, c, d, e) = (1, 2, 3, 4, 5)$ or $(2, 1, 3, 4, 5)$, and the smallest value is obtained by letting $(a, b, c, d, e) = (5, 4, 3, 2, 1)$ or $(4, 5, 3, 2, 1)$. In the former case the iterative average is $65/16$, and in the latter case the iterative average is $31/16$, so the desired difference is

$$\frac{65}{16} - \frac{31}{16} = \frac{34}{16} = \frac{17}{8}.$$

2005B 9. **(B)** The percentage of students getting 95 points is

$$100 - 10 - 25 - 20 - 15 = 30,$$

so the mean score on the exam is

$$0.10(70) + 0.25(80) + 0.20(85) + 0.15(90) + 0.30(95) = 86.$$

Since fewer than half of the scores were less than 85, and fewer than half of the scores were greater than 85, the median score is 85. The difference between the mean and the median score on this exam is $86 - 85 = 1$.

1999

9. **(D)** The next palindromes after 29792 are 29892, 29992, 30003, and 30103. The difference $30103 - 29792 = 311$ is too far to drive in three hours without exceeding the speed limit of 75 miles per hour. Ashley could have driven $30003 - 29792 = 211$ miles during the three hours for an average speed of $70\frac{1}{3}$ miles per hour.

2004A

10. **(C)** The sum of a set of integers is the product of the mean and the number of integers, and the median of a set of consecutive integers is the same as the mean. So the median must be $7^5/49 = 7^3$.

2010B

10. **Answer (B):** The average of the numbers is

$$\frac{1+2+\cdots+99+x}{100} = \frac{\frac{99\cdot 100}{2}+x}{100} = \frac{99\cdot 50+x}{100} = 100x.$$

This equation is equivalent to $9999x = (99\cdot 101)x = 99\cdot 50$, so $x = \frac{50}{101}$.

2018B

10. **Answer (D):** The list has $2018 - 10 = 2008$ entries that are not equal to the mode. Because the mode is unique, each of these 2008 entries can occur at most 9 times. There must be at least $\lceil \frac{2008}{9} \rceil = 224$ distinct values in the list that are different from the mode, because if there were fewer than this many such values, then the size of the list would be at most $9 \cdot (\lceil \frac{2008}{9} \rceil - 1) + 10 = 2017 < 2018$. (The ceiling function notation $\lceil x \rceil$ represents the least integer greater than or equal to x.) Therefore the least possible number of distinct values that can occur in the list is 225. One list satisfying the conditions of the problem contains 9 instances of each of the numbers 1 through 223, 10 instances of the number 224, and one instance of 225.