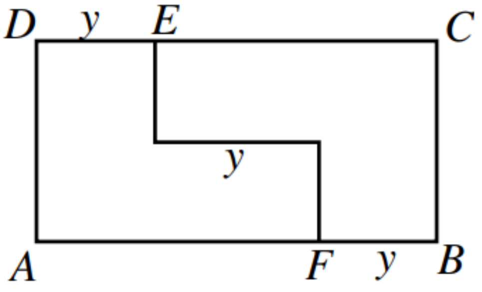
UNIT 1 EXERCISES 6-10

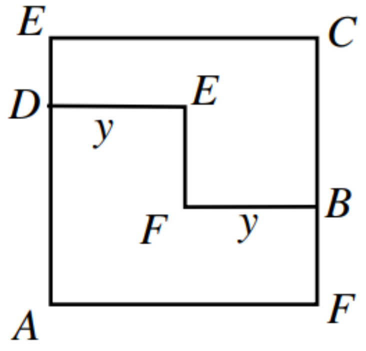
2D Geometry

2006A 6. **(A)** Let E represent the end of the cut on $\overline{DC}$, and let F represent the end of the cut on $\overline{AB}$. For a square to be formed, we must have

$$DE = y = FB \quad \text{and} \quad DE + y + FB = 18, \quad \text{so} \quad y = 6.$$

The rectangle that is formed by this cut is indeed a square, since the original rectangle has area $8 \cdot 18 = 144$, and the rectangle that is formed by this cut has a side of length $2 \cdot 6 = 12 = \sqrt{144}$.





2005A 7. **(C)** The symmetry of the figure implies that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles. So

$$BH = CE = \sqrt{BC^2 - BE^2} = \sqrt{50-1} = 7,$$

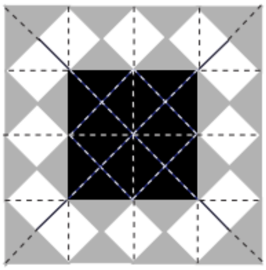
and $EH = BH - BE = 7 - 1 = 6$. Hence the square $EFGH$ has area $6^2 = 36$.

OR

As in the first solution, $BH = 7$. Now note that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles, so

$$\text{Area}(EFGH) = \text{Area}(ABCD) - 4\text{Area}(\triangle ABH) = 50 - 4\left(\tfrac{1}{2} \cdot 1 \cdot 7\right) = 36.$$

2002A 8. **(A)** Draw additional lines to cover the entire figure with congruent triangles. There are 24 triangles in the blue region, 24 in the white region, and 16 in the red region. Thus, $B = W$.



2004A 8. **(B)** Let x, y, and z be the areas of $\triangle ADE$, $\triangle BDC$, and $\triangle ABD$, respectively. The area of $\triangle ABE$ is $(1/2)(4)(8) = 16 = x + z$, and the area of $\triangle BAC$ is $(1/2)(4)(6) = 12 = y + z$. The requested difference is

$$x - y = (x+z) - (y+z) = 16 - 12 = 4.$$

2009A 8. **Answer (A):** Let the lengths of the shorter and longer side of each rectangle be x and y, respectively. The outer and inner squares have side lengths $y+x$ and $y-x$, respectively, and the ratio of their side lengths is $\sqrt{4}=2$. Therefore $y+x=2(y-x)$, from which $y=3x$.

2011B 8. **Answer (A):** The only parts of the track that are longer walking on the outside edge rather than the inside edge are the two semicircular portions. If the radius of the inner semicircle is r, then the difference in the lengths of the two paths is $2\pi(r+6)-2\pi r=12\pi$. Let x be her speed in meters per second. Then $36x=12\pi$, and $x=\frac{\pi}{3}$.

2016A 8. **Answer (D):** The diagonal of the rectangle from upper left to lower right divides the shaded region into four triangles. Two of them have a 1-unit horizontal base and altitude $\frac{1}{2}\cdot 5=2\frac{1}{2}$, and the other two have a 1-unit vertical base and altitude $\frac{1}{2}\cdot 8=4$. Therefore the total area is $2\cdot\frac{1}{2}\cdot 1\cdot 2\frac{1}{2}+2\cdot\frac{1}{2}\cdot 1\cdot 4=6\frac{1}{2}$.

2018A

8. **Answer (E):** The length of the base $\overline{DE}$ of $\triangle ADE$ is 4 times the length of the base of a small triangle, so the area of $\triangle ADE$ is $4^2 \cdot 1 = 16$. Therefore the area of $DBCE$ is the area of $\triangle ABC$ minus the area of $\triangle ADE$, which is $40 - 16 = 24$.

2013A

9. **Answer (C):** Because $\overline{EF}$ is parallel to $\overline{AB}$, it follows that $\triangle FEC$ is similar to $\triangle ABC$ and $FE = FC$. Thus half of the perimeter of $ADEF$ is $AF + FE = AF + FC = AC = 28$. The entire perimeter is 56.

2014B

9. **Answer (B):** By the Pythagorean Theorem, $AC = 5$. Because $5^2 + 12^2 = 13^2$, the converse of the Pythagorean Theorem applied to $\triangle DAC$ implies that $\angle DAC = 90°$. The area of $\triangle ABC$ is 6 and the area of $\triangle DAC$ is 30. Thus the area of the quadrilateral is $6 + 30 = 36$.

2016A 9. **Answer (E):** Let x be the common side length. Draw a diagonal between opposite corners of the unit square. The length of this diagonal is $\sqrt{2}$. The diagonal consists of two small-square diagonals and one small-square side length. Combining the previous two observations yields

$$2x\sqrt{2} + x = \sqrt{2}.$$

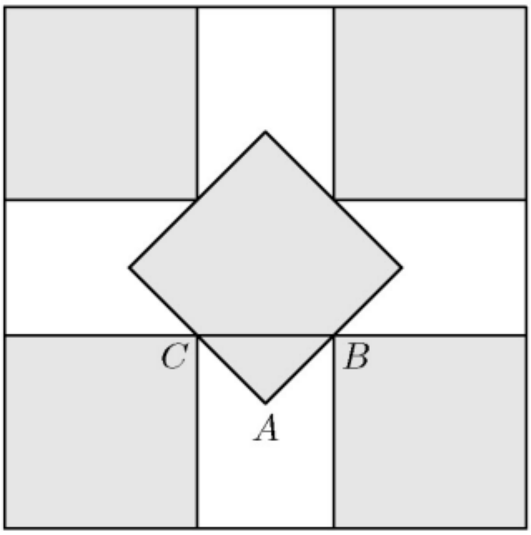
Solving this equation for x gives $x = \frac{4-\sqrt{2}}{7}$. The requested sum is $4 + 7 = 11$.

OR

Again let x be the common side length. Triangle ABC in the figure shown is a right triangle with sides $\frac{x}{2}$, $\frac{x}{2}$, and $1 - 2x$. By the Pythagorean Theorem,

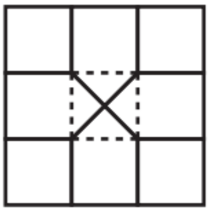
$$\left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = (1 - 2x)^2.$$

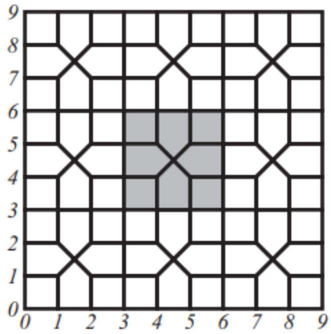
Solving this equation and noting that $x < \frac{1}{2}$ yields $x = \frac{4-\sqrt{2}}{7}$, as above.



2001 10. **(D)** The pattern shown at left is repeated in the plane. In fact, nine repetitions of it are shown in the statement of the problem. Note that four of the nine squres in the three-by-three square are not in the four pentagons that make up the three-by-three square. Therefore, the percentage of the plane that is enclosed by pentagons is

$$1 - \frac{4}{9} = \frac{5}{9} = 55\frac{5}{9}\%$$





2003B 10. (B) We may assume that one of the triangles is attached to side $\overline{AB}$. The second triangle can be attached to $\overline{BC}$ or $\overline{CD}$ to obtain two non-congruent figures. If the second triangle is attached to $\overline{AE}$ or to $\overline{DE}$, the figure can be reflected about the vertical axis of symmetry of the pentagon to obtain one of the two already counted. Thus the total is two.

2004B 10. **(A)** The area of the annulus is the difference between the areas of the two circles, which is $\pi b^2 - \pi c^2$. Because the tangent $\overline{XZ}$ is perpendicular to the radius $\overline{OZ}$, $b^2 - c^2 = a^2$, so the area is πa^2.

2009A 10. **Answer (C):** Let x be the length of $\overline{BD}$. By the triangle inequality on $\triangle BCD$, $5 + x > 17$, so $x > 12$. By the triangle inequality on $\triangle ABD$, $5 + 9 > x$, so $x < 14$. Since x must be an integer, $x = 13$.