UNIT 7 QUESTIONS 16-20

FOOTBALL

2010B 19. **Answer (E):** The Raiders' score was $a(1+r+r^2+r^3)$, where a is a positive integer and $r>1$. Because ar is also an integer, $r=m/n$ for relatively prime positive integers m and n with $m>n$. Moreover $ar^3=a\cdot\frac{m^3}{n^3}$ is an integer, so n^3 divides a. Let $a=n^3A$. Then the Raiders' score was $R=A(n^3+mn^2+m^2n+m^3)$, and the Wildcats' score was $R-1=a+(a+d)+(a+2d)+(a+3d)=4a+6d$ for some positive integer d. Because $A\geq 1$, the condition $R\leq 100$ implies that $n\leq 2$ and $m\leq 4$. The only possibilities are $(m,n)=(4,1),(3,2),(3,1),(4,1)$, or $(2,1)$. The corresponding values of R are, respectively, $85A$, $65A$, $40A$, and $15A$. In the first two cases $A=1$, and the corresponding values of $R-1$ are, respectively, $64=32+6d$ and $84=4+6d$. In neither case is d an integer. In the third case $40A=40a=4a+6d+1$ which is impossible in integers. In the last case $15a=4a+6d+1$, from which $11a=6d+1$. The only solution in positive integers for which $4a+6d\leq 100$ is $(a,d)=(5,9)$. Thus $R=5+10+20+40=75$, $R-1=5+14+23+32=74$, and the number of points scored in the first half was $5+10+5+14=34$.