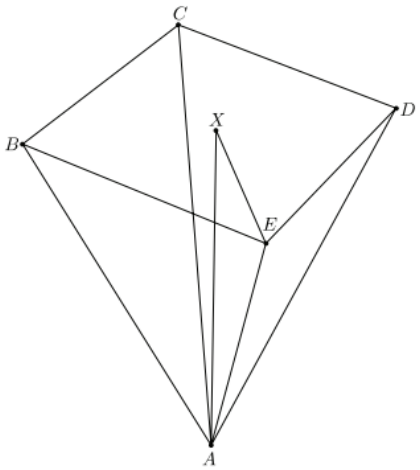
UNIT 11 QUESTIONS 16-20

SYSTEM

2014A 17. **Answer (A):** Connect the centers of the large sphere and the four small spheres at the top to form an inverted square pyramid as shown. Since $BCDE$ is a square of side 2, $EX = \sqrt{2}$. Also, $AE = 3$ and $\triangle AXE$ is a right triangle, so

$$AX = \sqrt{3^2 - \left(\sqrt{2}\right)^2} = \sqrt{7}.$$

The distance from the plane containing $BCDE$ to the top of the box is 1. Therefore the total height of the box is $2(1 + AX) = 2 + 2\sqrt{7}$.



2005B 19. **(E)** By the given conditions, it follows that $x > y$. Let $x = 10a + b$ and $y = 10b + a$, where $a > b$. Then

$$m^2 = x^2 - y^2 = (10a + b)^2 - (10b + a)^2 = 99a^2 - 99b^2 = 99(a^2 - b^2).$$

Since $99(a^2 - b^2)$ must be a perfect square,

$$a^2 - b^2 = (a + b)(a - b) = 11k^2,$$

for some positive integer k. Because a and b are distinct digits, we have $a - b \leq 9 - 1 = 8$ and $a + b \leq 9 + 8 = 17$. It follows that $a + b = 11$, $a - b = k^2$, and k is either 1 or 2.

If $k = 2$, then $(a, b) = (15/2, 7/2)$, which is impossible. Thus $k = 1$ and $(a, b) = (6, 5)$. This gives $x = 65$, $y = 56$, $m = 33$, and $x + y + m = 154$.