UNIT 6 EXERCISES 11-15

SPEED TIME

2002A

11. **(B)** Let t be the number of hours Mr. Bird must travel to arrive on time. Since three minutes is the same as 0.05 hours, $40(t+0.05) = 60(t-0.05)$. Thus,

$$40t + 2 = 60t - 3, \quad \text{so } t = 0.25.$$

The distance from his home to work is $40(0.25 + 0.05) = 12$ miles. Therefore, his average speed should be $12/0.25 = 48$ miles per hour.

OR

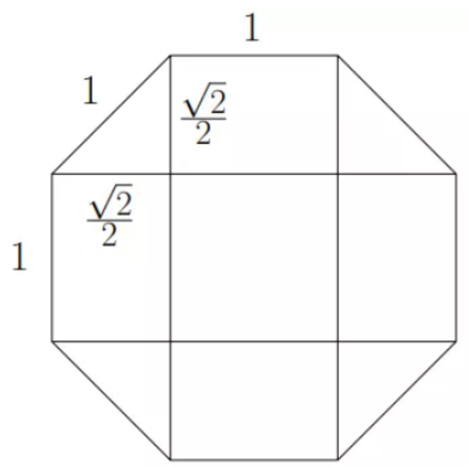
Let d be the distance from Mr. Bird's house to work, and let s be the desired average speed. Then the desired driving time is d/s. Since $d/60$ is three minutes too short and $d/40$ is three minutes too long, the desired time must be the average, so

$$\frac{d}{s} = \frac{1}{2}\left(\frac{d}{60} + \frac{d}{40}\right).$$

This implies that $s = 48$.

2011A

12. **Answer (A):** Assume the octagon's edge is 1. Then the corner triangles have hypotenuse 1 and thus legs $\frac{\sqrt{2}}{2}$ and area $\frac{1}{4}$ each; the four rectangles are 1 by $\frac{\sqrt{2}}{2}$ and have area $\frac{\sqrt{2}}{2}$ each, and the center square has area 1. The total area is $4 \cdot \frac{1}{4} + 4 \cdot \frac{\sqrt{2}}{2} + 1 = 2 + 2\sqrt{2}$. The probability that the dart hits the center square is $\frac{1}{2+2\sqrt{2}} = \frac{\sqrt{2}-1}{2}$.



2017A

12. **Answer (B):** Horse k will again be at the starting point after t minutes if and only if k is a divisor of t. Let $I(t)$ be the number of integers k with $1 \le k \le 10$ that divide t. Then $I(1) = 1$, $I(2) = 2$, $I(3) = 2$, $I(4) = 3$, $I(5) = 2$, $I(6) = 4$, $I(7) = 2$, $I(8) = 4$, $I(9) = 3$, $I(10) = 4$, $I(11) = 1$, and $I(12) = 5$. Thus $T = 12$ and the requested sum of digits is $1 + 2 = 3$.

2012A

13. **Answer (D):** Let the length of the lunch break be m minutes. Then the three painters each worked $480 - m$ minutes on Monday, the two helpers worked $372 - m$ minutes on Tuesday, and Paula worked $672 - m$ minutes on Wednesday. If Paula paints $p\%$ of the house per minute and her helpers paint a total of $h\%$ of the house per minute, then

$$\begin{aligned}(p + h)(480 - m) &= 50,\\ h(372 - m) &= 24, \text{and}\\ p(672 - m) &= 26.\end{aligned}$$

Adding the last two equations gives $672p + 372h - mp - mh = 50$, and subtracting this equation from the first one gives $108h - 192p = 0$, so $h = \frac{16p}{9}$. Substitution into the first equation then leads to the system

$$\begin{aligned}\frac{25p}{9}(480 - m) &= 50,\\ p(672 - m) &= 26.\end{aligned}$$

The solution of this system is $p = \frac{1}{24}$ and $m = 48$. Note that $h = \frac{2}{27}$.

2017A 13. **Answer (B):** Let d be the requested distance in miles, and suppose that Sharon usually drives at speed r in miles per hour. Then $\frac{d}{r} = 3$. The total time in hours for Sharon's trip with the snowstorm is then $\frac{\frac{1}{3}d}{r} + \frac{\frac{2}{3}d}{r-20} = \frac{23}{5}$. Because $\frac{d}{r} = 3$, this reduces to

$$1 + \frac{\frac{2}{3}}{\frac{r}{d} - \frac{20}{d}} = 1 + \frac{\frac{2}{3}}{\frac{1}{3} - \frac{20}{d}} = \frac{23}{5}.$$

Solving for d gives $d = 135$.

OR

The last $\frac{2}{3}$ of the drive takes $276 - \frac{1}{3} \cdot 180 = 216$ minutes, which is $\frac{216}{60}$ hours. If r is the original speed in miles per hour, then $\frac{2}{3}$ of the distance is both $2r$ and $\frac{216}{60} \cdot (r - 20)$. Setting these expressions equal and solving yields $r = 45$. Therefore the original speed is 45 miles per hour, and the requested distance is $3 \cdot 45 = 135$ miles.

2004A 15. **(C)** When they first meet, they have run a combined distance equal to half the length of the track. Between their first and second meetings, they run a combined distance equal to the full length of the track. Because Brenda runs at a constant speed and runs 100 meters before their first meeting, she runs $2(100) = 200$ meters between their first and second meetings. Therefore the length of the track is $200 + 150 = 350$ meters.