UNIT 24 EXERCISES 11-15

TRIG

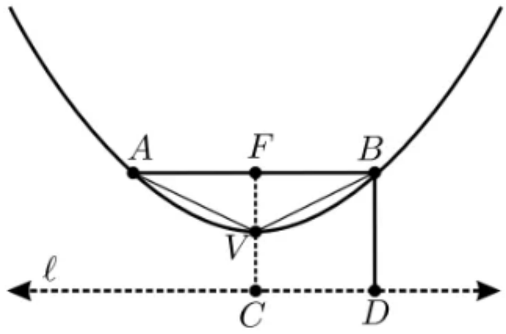
2010B

13. **Answer (C):** The maximum value for $\cos x$ and $\sin x$ is 1; hence $\cos(2A-B) = 1$ and $\sin(A+B) = 1$. Therefore $2A - B = 0°$ and $A + B = 90°$, and solving gives $A = 30°$ and $B = 60°$. Hence $\triangle ABC$ is a $30-60-90°$ right triangle and $BC = 2$.

2011B

14. **Answer (D):** Let ℓ be the directrix of the parabola, and let C and D be the projections of F and B onto ℓ, respectively. For any point in the parabola, its distance to F and to ℓ are the same. Because V and B are on the parabola, it follows that $p = FV = VC$ and $2p = FC = BD = FB$. By the Pythagorean Theorem, $VB = \sqrt{FV^2 + FB^2} = \sqrt{5}p$, and thus $\cos(\angle FVB) = \frac{FV}{VB} = \frac{p}{\sqrt{5}p} = \frac{\sqrt{5}}{5}$. Because $\angle AVB = 2(\angle FVB)$, it follows that

$$\cos(\angle AVB) = 2\cos^2(\angle FVB) - 1 = 2\left(\frac{\sqrt{5}}{5}\right)^2 - 1 = \frac{2}{5} - 1 = -\frac{3}{5}.$$



OR

Establish as in the first solution that $FV = p$, $FB = 2p$, and $VB = \sqrt{5}p$. Then $AB = 2 \cdot FB = 4p$, and by the Law of Cosines applied to $\triangle ABV$,

$$\cos \angle AVB = \frac{VA^2 + VB^2 - AB^2}{2(VA)(VB)} = \frac{5p^2 + 5p^2 - 16p^2}{2(5p^2)} = -\frac{3}{5}.$$

Note: The segment AB is called the *latus rectum*.

1999

15. **(E)** From the identity $1 + \tan^2 x = \sec^2 x$ it follows that $1 = \sec^2 x - \tan^2 x = (\sec x - \tan x)(\sec x + \tan x) = 2(\sec x + \tan x)$, so $\sec x + \tan x = 0.5$.

OR

The given relation can be written as $\dfrac{1 - \sin x}{\cos x} = 2$. Squaring both sides yields $\dfrac{(1-\sin x)^2}{1 - \sin^2 x} = 4$, hence $\dfrac{1 - \sin x}{1 + \sin x} = 4$. It follows that $\sin x = -\dfrac{3}{5}$ and that

$$\cos x = \frac{1 - \sin x}{2} = \frac{1 - (-3/5)}{2} = \frac{4}{5}.$$

Thus $\sec x + \tan x = \frac{5}{4} - \frac{3}{4} = 0.5$.

2006A

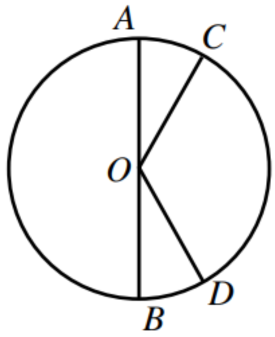
15. **(A)** Because $\cos x = 0$ and $\cos(x+z) = 1/2$, it follows that $x = m\pi/2$ for some odd integer m and $x + z = 2n\pi \pm \pi/3$ for some integer n. Therefore

$$z = 2n\pi - \frac{m\pi}{2} \pm \frac{\pi}{3} = k\pi + \frac{\pi}{2} \pm \frac{\pi}{3}$$

for some integer k. The smallest value of k that yields a positive value for z is 0, and the smallest positive value of z is $\pi/2 - \pi/3 = \pi/6$.

OR

Let O denote the center of the unit circle. Because $\cos x = 0$, the terminal side of an angle of measure x, measured counterclockwise from the positive x-axis, intersects the circle at $A = (0, 1)$ or $B = (0, -1)$.



Because $\cos(x+z) = 1/2$, the terminal side of an angle of measure $x+z$ intersects the circle at $C = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $D = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Thus all angles of positive measure $z = (x+z) - x$ can be measured counterclockwise from either $\overline{OA}$ or $\overline{OB}$ to either $\overline{OC}$ or $\overline{OD}$. The smallest such angle is $\angle BOD$, which has measure $\pi/6$ and is attained, for example, when $x = -\pi/2$ and $x + z = -\pi/3$.

2017A

15. **Answer (D):** For $0 < x < \frac{\pi}{2}$ all three terms are positive, and $f(x)$ is undefined when $x = \frac{\pi}{2}$. For $\frac{\pi}{2} < x < \frac{3\pi}{4}$, the term $3\tan x$ is less than -3 and dominates the other two terms, so $f(x) < 0$ there. For $\frac{3\pi}{4} \le x < \pi$, $|\cos(x)| \ge |\sin(x)|$ and $\cos x$ and $\tan x$ are negative, so $\sin x + 2\cos x + 3\tan x < 0$. Therefore there is no positive solution of $f(x) = 0$ for $x < \pi$. Because the range of f includes all values between $f(\pi) = -2 < 0$ and $f(\frac{5\pi}{4}) = -\frac{3}{2}\sqrt{2} + 3 > -1.5 \cdot 1.5 + 3 > 0$ on the interval $[\pi, \frac{5\pi}{4}]$, the smallest positive solution of $f(x) = 0$ lies between π and $\frac{5\pi}{4}$. Because $\pi > 3$ and $\frac{5\pi}{4} < 4$, the requested interval is $(3, 4)$.