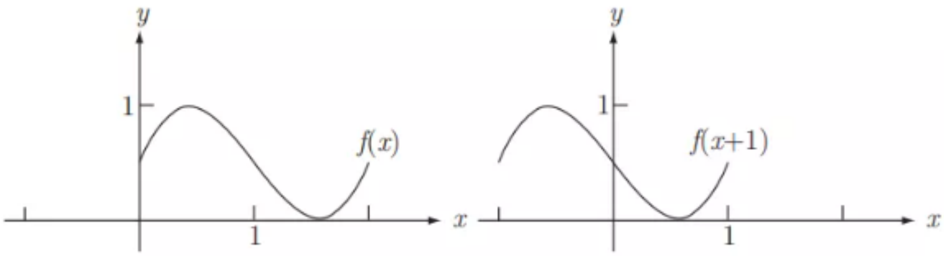
UNIT 23 EXERCISES 11-15

FUNCTIONS

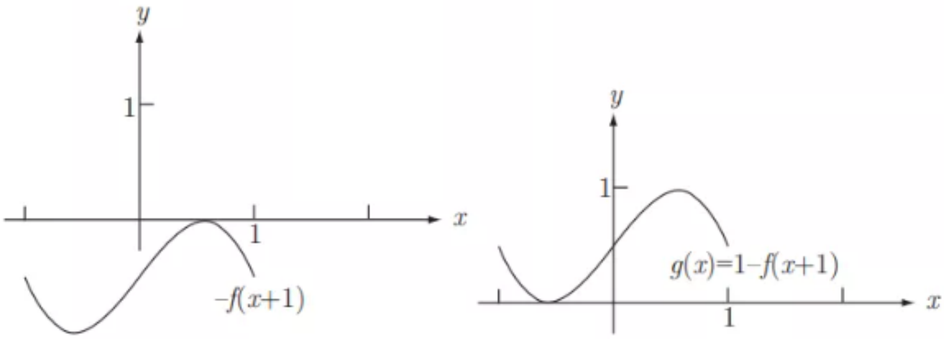
2008A

12. **Answer (B):** Because the domain of f is $[0,2]$, $f(x+1)$ is defined for $0 \le x+1 \le 2$, or $-1 \le x \le 1$. Thus $g(x)$ is also defined for $-1 \le x \le 1$, so its domain is $[-1,1]$. Because the range of f is $[0,1]$, the values of $f(x+1)$ are all the numbers between 0 and 1, inclusive. Thus the values of $g(x)$ are all the numbers between $1-0=1$ and $1-1=0$, inclusive, so the range of g is $[0,1]$.

OR



The graph of $y = f(x+1)$ is obtained by shifting the graph of $y = f(x)$ one unit to the left. The graph of $y = -f(x+1)$ is obtained by reflecting the graph of $y = f(x+1)$ across the x-axis. The graph of $y = g(x) = 1 - f(x+1)$ is obtained by shifting the graph of $y = -f(x+1)$ up one unit. As the figures illustrate, the domain and range of g are $[-1,1]$ and $[0,1]$, respectively.



2004B

13. **(A)** Since $f(f^{-1}(x)) = x$, it follows that $a(bx+a)+b = x$, so $ab = 1$ and $a^2 + b = 0$. Hence $a = b = -1$, so $a + b = -2$.

2002A 14. **(D)** We have

$$N = \log_{2002} 11^2 + \log_{2002} 13^2 + \log_{2002} 14^2 = \log_{2002} 11^2 \cdot 13^2 \cdot 14^2 = \log_{2002} (11 \cdot 13 \cdot 1$$

Simplifying gives

$$N = \log_{2002} (11 \cdot 13 \cdot 14)^2 = \log_{2002} 2002^2 = 2.$$

1999 15. **(E)** From the identity $1 + \tan^2 x = \sec^2 x$ it follows that $1 = \sec^2 x - \tan^2 x = (\sec x - \tan x)(\sec x + \tan x) = 2(\sec x + \tan x)$, so $\sec x + \tan x = 0.5$.

OR

The given relation can be written as $\dfrac{1 - \sin x}{\cos x} = 2$. Squaring both sides yields $\dfrac{(1 - \sin x)^2}{1 - \sin^2 x} = 4$, hence $\dfrac{1 - \sin x}{1 + \sin x} = 4$. It follows that $\sin x = -\dfrac{3}{5}$ and that

$$\cos x = \frac{1 - \sin x}{2} = \frac{1 - (-3/5)}{2} = \frac{4}{5}.$$

Thus $\sec x + \tan x = \frac{5}{4} - \frac{3}{4} = 0.5$.