UNIT 22 EXERCISES 11-15

QUAD/POLY

2006A

11. **(C)** The equation $(x + y)^2 = x^2 + y^2$ is equivalent to $x^2 + 2xy + y^2 = x^2 + y^2$, which reduces to $xy = 0$. Thus the graph of the equation consists of the two lines that are the coordinate axes.

2002A

12. **(B)** Let p and q be two primes that are roots of $x^2 - 63x + k = 0$. Then

$$x^2 - 63x + k = (x - p)(x - q) = x^2 - (p + q)x + p \cdot q,$$

so $p + q = 63$ and $p \cdot q = k$. Since 63 is odd, one of the primes must be 2 and the other 61. Thus, there is exactly one possible value for k, namely $k = p \cdot q = 2 \cdot 61 = 122$.

2005B

12. **(D)** Let r_1 and r_2 be the roots of $x^2+px+m=0$. Since the roots of $x^2+mx+n=0$ are $2r_1$ and $2r_2$, we have the following relationships:

$$m = r_1 r_2, \quad n = 4r_1 r_2, \quad p = -(r_1+r_2), \quad \text{and} \quad m = -2(r_1+r_2).$$

So

$$n = 4m, \quad p = \frac{1}{2}m, \quad \text{and} \quad \frac{n}{p} = \frac{4m}{\frac{1}{2}m} = 8.$$

OR

The roots of

$$\left(\frac{x}{2}\right)^2 + p\left(\frac{x}{2}\right) + m = 0$$

are twice those of $x^2+px+m=0$. Since the first equation is equivalent to $x^2+2px+4m=0$, we have

$$m = 2p \quad \text{and} \quad n = 4m, \quad \text{so} \quad \frac{n}{p} = 8.$$

2006B

12. **(D)** A parabola with the given equation and with vertex (p,p) must have equation $y = a(x-p)^2+p$. Because the y-intercept is $(0,-p)$ and $p \neq 0$, it follows that $a = -2/p$. Thus

$$y = -\frac{2}{p}(x^2 - 2px + p^2) + p = -\frac{2}{p}x^2 + 4x - p,$$

so $b = 4$.

2015B

12. **Answer (D):** If $(x-a)(x-b)+(x-b)(x-c)=0$, then $(x-b)(2x-(a+c))=0$, so the two roots are b and $\frac{a+c}{2}$. The maximum value of their sum is $9+\frac{8+7}{2}=16.5$.

2017B 12. **Answer (D):** The principal root of the equation $z^{12} = 64$ is

$$z = 64^{\frac{1}{12}} \cdot \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{2} \cdot \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right).$$

The 12 roots lie in the complex plane on the circle of radius $\sqrt{2}$ centered at the origin. The roots with positive real part make angles of 0, $\pm\frac{\pi}{6}$, and $\pm\frac{\pi}{3}$ with the positive real axis. When these five numbers are added, the imaginary parts cancel out and the sum is

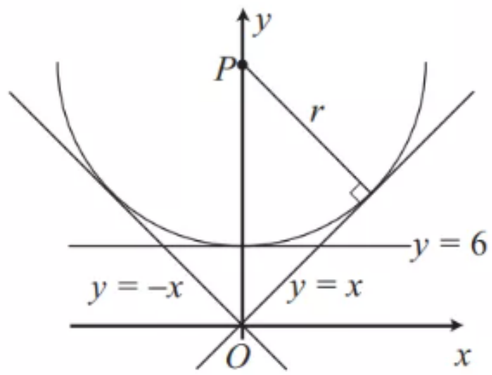
$$\sqrt{2} + 2\sqrt{2}\cdot\cos\frac{\pi}{6} + 2\sqrt{2}\cdot\cos\frac{\pi}{3} = \sqrt{2}\cdot(1+\sqrt{3}+1) = 2\sqrt{2}+\sqrt{6}.$$

2012B 13. **Answer (D):** The parabolas have no points in common if and only if the equation $x^2+ax+b = x^2+cx+d$ has no solution. This is true if and only if the lines with equations $y = ax+b$ and $y = cx+d$ are parallel, which happens if and only if $a = c$ and $b \neq d$. The probability that $a = c$ is $\frac{1}{6}$ and the probability that $b \neq d$ is $\frac{5}{6}$, so the probability that the two parabolas have a point in common is $1 - \frac{1}{6}\cdot\frac{5}{6} = \frac{31}{36}$.

2005B

14. **(E)** Let O denote the origin, P the center of the circle, and r the radius. A radius from the center to the point of tangency with the line $y = x$ forms a right triangle with hypotenuse $\overline{OP}$. This right triangle is isosceles since the line $y = x$ forms a $45°$ angle with the y-axis. So

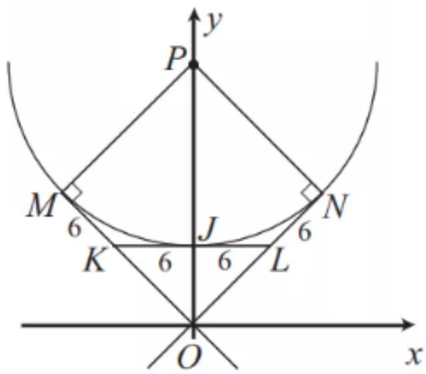
$$r\sqrt{2} = r + 6 \quad \text{and} \quad r = \frac{6}{\sqrt{2} - 1} = 6\sqrt{2} + 6.$$



OR

Let the line $y = -x$ intersect the circle and the line $y = 6$ at M and K, respectively, and let the line $y = x$ intersect the circle and the line $y = 6$ at N and L, respectively. Quadrilateral $PMON$ has four right angles and $MP = PN$, so $PMON$ is a square. In addition, $MK = KJ = 6$ and $KO = 6\sqrt{2}$. Hence

$$r = MO = MK + KO = 6 + 6\sqrt{2}.$$



2007A

14. **Answer (C):** If 45 is expressed as a product of five distinct integer factors, the absolute value of the product of any four is at least $|(-3)(-1)(1)(3)| = 9$, so no factor can have an absolute value greater than 5. Thus the factors of the given expression are five of the integers $\pm 1, \pm 3$, and ± 5. The product of all six of these is $-225 = (-5)(45)$, so the factors are -3, -1, 1, 3, and 5. The corresponding values of a, b, c, d, and e are 9, 7, 5, 3, and 1, and their sum is 25.