UNIT 20 EXERCISES 11-15

LOGIC

2010A

12. **Answer (D):** LeRoy and Chris cannot both be frogs, because their statements would be true and frogs lie. Also LeRoy and Chris cannot both be toads, because then their statements would be false, and toads tell the truth. Hence between LeRoy and Chris, exactly one must be a toad.

If Brian is a toad, then Mike must be a frog, but this is a contradiction as Mike's statement would then be true. Hence Brian is a frog, so Brian's statement must be false, and Mike must be a frog. Altogether there are 3 frogs: Brian, Mike, and either LeRoy or Chris.

2015A 13. **Answer (E):** Note that each of the 12 teams plays 11 games, so $\frac{12 \cdot 11}{2} = 66$ games are played in all. If every game ends in a draw, then each team will have a score of 11, so statement (E) is not true. Each of the other statements is true. Each of the games generates 2 points in the score list, regardless of its outcome, so the sum of the scores must be $66 \cdot 2 = 132$; thus (D) is true. Because the sum of an odd number of odd numbers plus any number of even numbers is odd, and 132 is even, there must be an even number of odd scores; thus (A) is true. Because there are 12 scores in all, there must also be an even number of even scores; thus (B) is true. Two teams cannot both have a score of 0 because the game between them must result in 1 point for each of them or 2 points for one of them; thus (C) is true.

1999 14. **(A)** Tina and Alina each sang either 5 or 6 times. If N denotes the number of songs sung by trios, then $3N = 4+5+5+7 = 21$ or $3N = 4+5+6+7 = 22$ or $3N = 4+6+6+7 = 23$. Since the girls sang as trios, the total must be a multiple of 3. Only 21 qualifies. Therefore, $N = 21/3 = 7$ is the number of songs the trios sang.

Challenge. Devise a schedule for the four girls so that each one sings the required number of songs.