UNIT 17 EXERCISES 11-15

ARITHMETIC

2002B 11. **(E)** The numbers $A-B$ and $A+B$ are both odd or both even. However, they are also both prime, so they must both be odd. Therefore, one of A and B is odd and the other even. Because A is a prime between $A-B$ and $A+B$, A must be the odd prime. Therefore, $B=2$, the only even prime. So $A-2$, A, and $A+2$ are consecutive odd primes and thus must be 3, 5, and 7. The sum of the four primes 2, 3, 5, and 7 is the prime number 17.

2003B 12. (D) Among five consecutive odd numbers, at least one is divisible by 3 and exactly one is divisible by 5, so the product is always divisible by 15. The cases $n=2$, $n=10$, and $n=12$ demonstrate that no larger common divisor is possible, since 15 is the greatest common divisor of $3\cdot5\cdot7\cdot9\cdot11$, $11\cdot13\cdot15\cdot17\cdot19$, and $13\cdot15\cdot17\cdot19\cdot21$.

2015A

14. **Answer (D):** By the change of base formula, $\dfrac{1}{\log_m n} = \log_n m$. Thus

$$1 = \frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = \log_a 2 + \log_a 3 + \log_a 4 = \log_a 24.$$

It follows that $a = 24$.

2011B 15. **Answer (D):** Factoring results in the following product of primes:

$$\begin{aligned}2^{24} - 1 &= (2^{12} - 1)(2^{12} + 1) = (2^6 - 1)(2^6 + 1)(2^4 + 1)(2^8 - 2^4 + 1)\\ &= 63 \cdot 65 \cdot 17 \cdot 241 = 3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17 \cdot 241.\end{aligned}$$

The two-digit integers that can be formed from these prime factors are:

$$\begin{gathered}17, \quad 3 \cdot 17 = 51, \quad 5 \cdot 17 = 85,\\ 13, \quad 3 \cdot 13 = 39, \quad 5 \cdot 13 = 65, \quad 7 \cdot 13 = 91,\\ 3 \cdot 7 = 21, \quad 5 \cdot 7 = 35, \quad 3 \cdot 3 \cdot 7 = 63,\\ 3 \cdot 5 = 15, \quad \text{and} \quad 3 \cdot 3 \cdot 5 = 45.\end{gathered}$$

Thus there are 12 positive two-digit factors.

2014A 15. **Answer (B):** Note that $abcba = a000a + b0b0 + c00$. Because $1 + 2 + \cdots + 9 = \frac{1}{2}(9 \cdot 10) = 45$, the sum of all integers of the form $a000a$ is 450 045. For each value of a there are $10 \cdot 10 = 100$ choices for b and c. Similarly, the sum of all integers of the form $b0b0$ is 45 450. For each value of b there are $9 \cdot 10 = 90$ choices for a and c. The sum of all integers of the form $c00$ is 4500, and for each c there are $9 \cdot 10 = 90$ choices for a and b. Thus $S = 100 \cdot 450\,045 + 90 \cdot 45\,450 + 90 \cdot 4500 = 45(1\,000\,100 + 90\,900 + 9000) = 45(1\,100\,000) = 49\,500\,000$. The sum of the digits is 18.