UNIT 11 EXERCISES 11-15

STATS MEAN

2004B 11. **(D)** Each score of 100 is 24 points above the mean, so the five scores of 100 represent a total of $(5)(24) = 120$ points above the mean. Those scores must be balanced by scores totaling 120 points below the mean. Since no student scored more than $76 - 60 = 16$ points below the mean, the number of other students in the class must be an integer no less than $120/16$. The smallest such integer is 8, so the number of students in the class is at least 13. Note that the conditions of the problem are met if 5 students score 100 and 8 score 61.

OR

If there are k students in the class, the sum of their scores is $76k$. If the five scores of 100 are excluded, the sum of the remaining scores is $76k - 500$. Since each student scored at least 60, the sum is at least $60(k - 5)$. Thus

$$76k - 500 \geq 60(k - 5),$$

so $k \geq 12.5$. Since k must be an integer, $k \geq 13$.

2014B 11. **Answer (E):** The numbers in the list have a sum of $11 \cdot 10 = 110$. The value of the 11th number is maximized when the sum of the first ten numbers is minimized subject to the following conditions.

- If the numbers are arranged in nondecreasing order, the sixth number is 9.
- The number 8 occurs either 2, 3, 4, or 5 times, and all other numbers occur fewer times.

If 8 occurs 5 times, the smallest possible sum of the first 10 numbers is

$$8 + 8 + 8 + 8 + 8 + 9 + 9 + 9 + 9 + 10 = 86.$$

If 8 occurs 4 times, the smallest possible sum of the first 10 numbers is

$$1 + 8 + 8 + 8 + 8 + 9 + 9 + 9 + 10 + 10 = 80.$$

If 8 occurs 3 times, the smallest possible sum of the first 10 numbers is

$$1 + 1 + 8 + 8 + 8 + 9 + 9 + 10 + 10 + 11 = 75.$$

If 8 occurs 2 times, the smallest possible sum of the first 10 numbers is

$$1 + 2 + 3 + 8 + 8 + 9 + 10 + 11 + 12 + 13 = 77.$$

Thus the largest possible value of the 11th number is $110 - 75 = 35$.

2007B 12. **Answer (C):** Let N be the number of students in the class. Then there are $0.1N$ juniors and $0.9N$ seniors. Let s be the score of each junior. The scores totaled $84N = 83(0.9N) + s(0.1N)$, so

$$s = \frac{84N - 83(0.9N)}{0.1N} = 93.$$

Note: In this problem, we could assume that the class has one junior and nine seniors. Then

$$9 \cdot 83 + s = 10 \cdot 84 = 9 \cdot 84 + 84 \quad \text{and} \quad s = 9(84 - 83) + 84 = 93.$$

1999

14. **(A)** Tina and Alina each sang either 5 or 6 times. If N denotes the number of songs sung by trios, then $3N = 4+5+5+7 = 21$ or $3N = 4+5+6+7 = 22$ or $3N = 4+6+6+7 = 23$. Since the girls sang as trios, the total must be a multiple of 3. Only 21 qualifies. Therefore, $N = 21/3 = 7$ is the number of songs the trios sang.

Challenge. Devise a schedule for the four girls so that each one sings the required number of songs.

2002A

15. **(D)** The values 6, 6, 6, 8, 8, 8, 8, 14 satisfy the requirements of the problem, so the answer is at least 14. If the largest number were 15, the collection would have the ordered form 7, __, __, 8, 8, __, __, 15. But $7+8+8+15 = 38$, and a mean of 8 implies that the sum of all values is 64. In this case, the four missing values would sum to $64-38 = 26$, and their average value would be 6.5. This implies that at least one would be less than 7, which is a contradiction. Therefore, the largest integer that can be in the set is 14.

2007A

15. **Answer (E):** The mean of the augmented set is $(28+n)/5$. If $n < 6$, the median of that set is 6, so $28+n = 5 \cdot 6$, and $n = 2$. If $6 < n < 9$, the median is n, so $28+n = 5n$, and $n = 7$. If $n > 9$, the median is 9, so $28+n = 5 \cdot 9$, and $n = 17$. Thus the sum of all possible values of n is $2+7+17 = 26$.