UNIT 21 EXERCISES 1-5

LOGIC

1999

2. **(A)** Triangles with side lengths of $1, 1, 1$ and $2, 2, 2$ are equilateral and not congruent, so (A) is false. Statement (B) is true since all triangles are convex. Statements (C) and (E) are true since each interior angle of an equilateral triangle measures $60°$. Furthermore, all three sides of an equilateral triangle have the same length, so (D) is also true.

2017A

3. **Answer (B):** The given statement is logically equivalent to its contrapositive: If a student did not receive an A on the exam, then the student did not get all the multiple choice questions right, which means that he got at least one of them wrong. None of the other statements follows logically from the given implication; the teacher made no promises concerning students who did not get all the multiple choice questions right. In particular, a statement does not imply its inverse or its converse; and the negation of the statement that Lewis got all the questions right is not the statement that he got all the questions wrong.

2004A   4. **(E)** Bertha has $30 - 6 = 24$ granddaughters, none of whom have any daughters. The granddaughters are the children of $24/6 = 4$ of Bertha's daughters, so the number of women having no daughters is $30 - 4 = 26$.

2015B   4. **Answer (B):** Marta finished 6th, so Jack finished 5th. Therefore Todd finished 3rd and Rand finished 2nd. Because Hikmet was 6 places behind Rand, it was Hikmet who finished 8th. (David finished 10th.)

2018A   4. **Answer (D):** Because the statements of Alice, Bob, and Charlie are all incorrect, the actual distance d satisfies $d < 6$, $d > 5$, and $d > 4$. Hence the actual distance lies in the interval $(5,6)$.

2008A   5. **Answer (B):** Because

$$\frac{2x}{3} - \frac{x}{6} = \frac{x}{2}$$

is an integer, x must be even. The case $x = 4$ shows that x is not necessarily a multiple of 3 and that none of the other statements must be true.

2015A   5. **Answer (D):**   As long as x and y and their rounded values are positive, rounding the dividend x up in a division problem $\frac{x}{y}$ makes the answer larger, and rounding x down makes the answer smaller. Similarly, rounding the divisor y up makes the answer smaller, and rounding y down makes the answer larger. In a subtraction problem $x - y$, rounding x up or rounding y down increases the answer, and rounding x down or rounding y up decreases it. Only in choice (D) do all the roundings contribute to increasing the answer. In the other situations, the estimate may be larger or smaller than the exact value, depending on the the amount by which each number is rounded and their values. In particular, the rounding may make the answer smaller.

For (A), $\frac{999\,999}{900} - 490 > \frac{1\,000\,000}{1\,000} - 500$.

For (B), $\frac{999\,999}{900} - 510 > \frac{1\,000\,000}{1\,000} - 500$.

For (C), $\frac{999\,999}{1\,001} - 490 > \frac{1\,000\,000}{1\,000} - 500$.

For (E), $\frac{1\,000\,001}{1\,001} - 490 > \frac{1\,000\,000}{1\,000} - 500$.

2016A   5. **Answer (E):**   A counterexample must satisfy the hypothesis of being an even integer greater than 2 but fail to satisfy the conclusion that it can be written as the sum of two prime numbers.