UNIT 19 EXERCISES 1-5

ALGEBRA WORD PROBLEMS

2000

1. **Answer (E):** Factor 2001 into primes to get $2001 = 3 \cdot 23 \cdot 29$. The largest possible sum of three distinct factors whose product is the one which combines the two largest prime factors, namely $I = 23 \cdot 29 = 667$, $M = 3$, and $O = 1$, so the largest possible sum is $1 + 3 + 667 = 671$.

2003A

1. (D) Each even counting number, beginning with 2, is one more than the preceding odd counting number. Therefore the difference is $(1)(2003) = 2003$.

2001

2. **(E)** Suppose $N = 10a + b$. Then $10a + b = ab + (a + b)$. It follows that $9a = ab$, which implies that $b = 9$, since $a \neq 0$.

2004B

2. **(D)** If $d \neq 0$, the value of the expression can be increased by interchanging 0 with the value of d. Therefore the maximum value must occur when $d = 0$. If $a = 1$, the value is c, which is 2 or 3. If $b = 1$, the value is $c \cdot a = 6$. If $c = 1$, the value is a^b, which is $2^3 = 8$ or $3^2 = 9$. Thus the maximum value is 9.

2008B

2. **Answer (B):** The two sums are $1 + 10 + 17 + 22 = 50$ and $4+9+16+25 = 54$, so the positive difference between the sums is $54 - 50 = 4$.

Query: If a different 4×4 block of dates had been chosen, the answer would be unchanged. Why?

1	2	3	4
11	10	9	8
15	16	17	18
25	24	23	22

2016B

2. **Answer (A):** The harmonic mean of 1 and 2016 is

$$\frac{2 \cdot 1 \cdot 2016}{1 + 2016} = 2 \cdot \frac{2016}{2017} \approx 2 \cdot 1 = 2.$$

2002B

3. **(B)** If $n \geq 4$, then

$$n^2 - 3n + 2 = (n-1)(n-2)$$

is the product of two integers greater than 1, and thus is not prime. For $n = 1$, 2, and 3 we have, respectively,

$$(1-1)(1-2) = 0, \quad (2-1)(2-2) = 0, \quad \text{and} \quad (3-1)(3-2) = 2.$$

Therefore, $n^2 - 3n + 2$ is prime only when $n = 3$.

2003A

5. (E) Since the last two digits of $AMC10$ and $AMC12$ sum to 22, we have

$$AMC + AMC = 2(AMC) = 1234.$$

Hence $AMC = 617$, so $A = 6$, $M = 1$, $C = 7$, and $A + M + C = 6 + 1 + 7 = 14$.

2004A 3. **(B)** The value of $x = 100 - 2y$ is a positive integer for each positive integer y with $1 \le y \le 49$.

2008B 3. **Answer (C):** A single player can receive the largest possible salary only when the other 20 players on the team are each receiving the minimum salary of \$15,000. Thus the maximum salary for any player is $\$700{,}000 - 20 \cdot \$15{,}000 = \$400{,}000$.

2014A 3. **Answer (B):** If Ralph passed the orange house first, then because the blue and yellow houses are not neighbors, the house color ordering must be orange, blue, red, yellow. If Ralph passed the blue house first, then there are 2 possible placements for the yellow house, and each choice determines the placement of the orange and red houses. These 2 house color orderings are blue, orange, yellow, red, and blue, orange, red, yellow. There are 3 possible orderings for the colored houses.

1999 4. **(A)** A number one less than a multiple of 5 is has a units digit of 4 or 9. A number whose units digit is 4 cannot be one greater than a multiple of 4. Thus, it is sufficient to examine the numbers of the form $10d + 9$ where d is one of the ten digits. Of these, only $9, 29, 49, 69$ and 89 are one greater than a multiple of 4. Among these, only 29 and 89 are prime and their sum is 118.

2005A 4. **(A)** If Dave buys seven windows separately he will purchase six and receive one free, for a cost of \$600. If Doug buys eight windows separately, he will purchase seven and receive one free, for a total cost of \$700. The total cost to Dave and Doug purchasing separately will be \$1300. If they purchase fifteen windows together, they will need to purchase only 12 windows, for a cost of \$1200, and will receive 3 free. This will result in a savings of \$100.

2010A 4. **Answer (D):** Choice **(D)** may be written as $-\frac{1}{x}$. If x is negative, choice **(D)** is positive. To see that the other choices need not be positive, let $x = -1$ and then

$$\textbf{(A)}\,\frac{-1}{|-1|} = -1,$$
$$\textbf{(B)}\,-(-1)^2 = -1,$$
$$\textbf{(C)}\,-2^{-1} = -\frac{1}{2},$$
$$\textbf{(E)}\,\sqrt[3]{-1} = -1.$$

2001 5. **(D)** Note that

$$1 \cdot 3 \cdots 9999 = \frac{1 \cdot 2 \cdot 3 \cdots 9999 \cdot 10000}{2 \cdot 4 \cdots 10000} = \frac{10000!}{2^{5000} \cdot 1 \cdot 2 \cdots 5000} = \frac{10000!}{2^{5000} \cdot 5000!}$$

2007B 5. **Answer (D):** Sarah will receive 4.5 points for the three questions she leaves unanswered, so she must earn at least $100 - 4.5 = 95.5$ points on the first 22 problems. Because

$$15 < \frac{95.5}{6} < 16,$$

she must solve at least 16 of the first 22 problems correctly. This would give her a score of 100.5.

2010A

5. **Answer (C):** The second place archer could score a maximum of $50 \cdot 10 = 500$ points with the remaining shots. Therefore Chelsea needs to score more than $500 - 50 = 450$ points to guarantee victory. If Chelsea's next n shots will score $10n$ points, her remaining $50 - n$ shots will score at least $4(50 - n)$ points. To guarantee victory,

$$\begin{aligned} 10 \cdot n + 4 \cdot (50 - n) &> 450 \\ 6n + 200 &> 450 \\ n &> 41\frac{2}{3}. \end{aligned}$$

Therefore Chelsea needs at least 42 bullseyes to guarantee victory.

OR

If Chelsea does not make a bullseye, the maximum number of points her opponents could gain per shot would be $10 - 4 = 6$. Chelsea must make enough bullseyes to prevent her opponents from gaining 50 points. Because $8 \cdot 6 < 50 < 9 \cdot 6$, the most non-bullseyes she can afford to score is 8, leaving $50 - 8 = 42$ bullseyes needed to guarantee her victory.

2010B

5. **Answer (D):** The correct answer was $1-(2-(3-(4+e))) = 1-2+3-4-e = -2-e$. Larry's answer was $1-2-3-4+e = -8+e$. Therefore $-2-e = -8+e$, so $e = 3$.