

UNIT 17 EXERCISES 1-5

CALCULATE VALUE

1999

1. (E) Pairing the first two terms, the next two terms, etc. yields

$$\begin{aligned}1 - 2 + 3 - 4 + \cdots - 98 + 99 &= \\(1 - 2) + (3 - 4) + \cdots + (97 - 98) + 99 &= \\-1 - 1 - 1 - \cdots - 1 + 99 &= 50,\end{aligned}$$

since there are 49 of the -1 's.

OR

$$\begin{aligned}1 - 2 + 3 - 4 + \cdots - 98 + 99 &= \\1 + [(-2 + 3) + (-4 + 5) + \cdots + (-98 + 99)] &= \\1 + [1 + 1 + \cdots + 1] &= 1 + 49 = 50.\end{aligned}$$

- 2006B 1. **(C)** Because

$$(-1)^k = \begin{cases} 1, & \text{if } k \text{ is even,} \\ -1, & \text{if } k \text{ is odd,} \end{cases}$$

the sum can be written as

$$(-1 + 1) + (-1 + 1) + \cdots + (-1 + 1) = 0 + 0 + \cdots + 0 = 0.$$

- 2010A 1. **Answer (C):** Distributing the negative signs gives

$$\begin{aligned} & (20 - (2010 - 201)) + (2010 - (201 - 20)) \\ &= 20 - 2010 + 201 + 2010 - 201 + 20 \\ &= 40. \end{aligned}$$

- 2011B 1. **Answer (C):** The given expression is equal to

$$\frac{12}{9} - \frac{9}{12} = \frac{4}{3} - \frac{3}{4} = \frac{16 - 9}{12} = \frac{7}{12}.$$

- 2012A 1. **Answer (E):** The distance from -2 to -6 is $|(-6) - (-2)| = 4$ units. The distance from -6 to 5 is $|5 - (-6)| = 11$ units. Altogether the bug crawls $4 + 11 = 15$ units.

2015A 1. **Answer (C):**

$$(1 - 1 + 25 + 0)^{-1} \times 5 = \frac{1}{25} \times 5 = \frac{1}{5}$$

2015B 1. **Answer (C):**

$$2 - (-2)^{-2} = 2 - \frac{1}{(-2)^2} = 2 - \frac{1}{4} = \frac{7}{4}$$

2016A 1. **Answer (B):**

$$\frac{11! - 10!}{9!} = \frac{10! \cdot (11 - 1)}{9!} = \frac{10 \cdot 9! \cdot 10}{9!} = 100$$

2000 2. **Answer (A):** $2000(2000^{2000}) = (2000^1)(2000^{2000}) = 2000^{1+2000} = 2000^{2001}$.
All the other options are greater than 2000^{2001} .

2008A 2. **Answer (A):** Note that

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}.$$

The reciprocal of $\frac{7}{6}$ is $\frac{6}{7}$.

- 2009A 2. **Answer (C):** Simplifying the expression,

$$1 + \frac{1}{1 + \frac{1}{1+1}} = 1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = \frac{5}{3}.$$

- 2016A 2. **Answer (C):** The equation can be written $10^x \cdot (10^2)^{2x} = (10^3)^5$ or $10^x \cdot 10^{4x} = 10^{15}$. Thus $10^{5x} = 10^{15}$, so $5x = 15$ and $x = 3$.

- 2002A 3. **(B)** No matter how the exponentiations are performed, 2^{2^2} always gives 16. Depending on which exponentiation is done last, we have

$$\left(2^{2^2}\right)^2 = 256, \quad 2^{\left(2^{2^2}\right)} = 65,536, \quad \text{or} \quad \left(2^2\right)^{\left(2^2\right)} = 256,$$

so there is one other possible value.

- 2016B 3. **Answer (D):**

$$\begin{aligned} & \left| \left| -2016 - (-2016) \right| - \left| -2016 \right| \right| - (-2016) \\ &= \left| \left| 2016 + 2016 \right| - 2016 \right| + 2016 = 2016 + 2016 = 4032 \end{aligned}$$

- 2013A 4. **Answer (C):** Factoring 2^{2012} from each of the terms and simplifying gives

$$\frac{2^{2012}(2^2 + 1)}{2^{2012}(2^2 - 1)} = \frac{4 + 1}{4 - 1} = \frac{5}{3}.$$

2011B 5. Answer (A):

Because N is divisible by 3, 4, and 5, the prime factorization of N must contain one 3, two 2s, and one 5. Furthermore $2^2 \cdot 3 \cdot 5 = 60$ is divisible by every integer less than 7. Therefore the numbers with this property are precisely the positive multiples of 60. The second smallest positive multiple of 60 is 120, and the sum of its digits is 3.