

UNIT 11 EXERCISES 1-5

STATS MEAN

2002B 1. **(A)** The number M is equal to

$$\frac{1}{9}(9+99+999+\dots+999,999,999) = 1+11+111+\dots+111,111,111 = 123,456,789.$$

The number M does not contain the digit 0.

2014A 1. **Answer (C):** Note that

$$10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10} \right)^{-1} = 10 \cdot \left(\frac{8}{10} \right)^{-1} = \frac{25}{2}.$$

- 2004A 2. **(C)** The 8 unanswered problems are worth $(2.5)(8) = 20$ points, so Charlyn must earn at least 80 additional points. The smallest multiple of 6 that is at least 80 is $(6)(14) = 84$, so Charlyn must have at least 14 correct answers.

- 2011B 2. **Answer (E):** The sum of her first 5 test scores is 385, yielding an average of 77. To raise her average to 80, her 6th test score must be the difference between $6 \cdot 80 = 480$ and 385, which is 95.

- 2015A 3. **Answer (E):** The sum of the 14 test scores was $14 \cdot 80 = 1120$. The sum of all 15 test scores was $15 \cdot 81 = 1215$. Therefore Payton's score was $1215 - 1120 = 95$.

OR

To bring the average up to 81, the total must include 1 more point for each of the 14 students, in addition to 81 points for Payton. Therefore Payton's score was $81 + 14 = 95$.

- 2001 4. **(D)** Since the median is 5, we can write the three numbers as x , 5, and y , where

$$\frac{1}{3}(x + 5 + y) = x + 10 \text{ and } \frac{1}{3}(x + 5 + y) + 15 = y.$$

If we add these equations, we get

$$\frac{2}{3}(x + 5 + y) + 15 = x + y + 10$$

and solving for $x + y$ gives $x + y = 25$. Hence the sum of the numbers $x + y + 5 = 30$.

OR

Let m be the mean of the three numbers. Then the least of the numbers is $m - 10$ and the greatest is $m + 15$. The middle of the three numbers is the median, 5. So

$$\frac{1}{3}((m - 10) + 5 + (m + 15)) = m$$

and $m = 10$. Hence, the sum of the three numbers is $3(10) = 30$.

- 2011A 4. **Answer (E):** Because $161 = 23 \cdot 7$, the only two digit factor of 161 is 23. The correct product must have been $32 \cdot 7 = 224$.

- 2016A 4. **Answer (D):** The mean of the data values is

$$\frac{60 + 100 + x + 40 + 50 + 200 + 90}{7} = \frac{x + 540}{7} = x.$$

Solving this equation for x gives $x = 90$. Thus the data in nondecreasing order are 40, 50, 60, 90, 90, 100, 200, so the median is 90 and the mode is 90, as required.

- 2005A 5. **(B)** The sum of the 50 numbers is $20 \cdot 30 + 30 \cdot 20 = 1200$. Their average is $1200/50 = 24$.

- 2014A 5. **Answer (C):** Because over 50% of the students scored 90 or lower, and over 50% of the students scored 90 or higher, the median score is 90. The mean score is

$$\frac{10}{100} \cdot 70 + \frac{35}{100} \cdot 80 + \frac{30}{100} \cdot 90 + \frac{25}{100} \cdot 100 = 87,$$

for a difference of $90 - 87 = 3$.

- 2017B 5. **Answer (B):** Because 1.5 times the interquartile range for this data set is $1.5 \cdot (43 - 33) = 15$, outliers are data values less than $33 - 15 = 18$ or greater than $43 + 15 = 58$. Only the value 6 meets this condition, so there is 1 outlier.