UNIT 10 EXERCISES 1-5

COMBINATIONS

2018A

3. **Answer (E):** There are 4 choices for the periods in which the mathematics courses can be taken: periods 1, 3, 5; periods 1, 3, 6; periods 1, 4, 6; and periods 2, 4, 6. Each choice of periods allows $3! = 6$ ways to order the 3 mathematics courses. Therefore there are $4 \cdot 6 = 24$ ways of arranging a schedule.

2008B

5. **Answer (C):** The total cost of the carnations must be an even number of dollars. The total number of dollars spent is the even number 50, so the number of roses purchased must also be even. In addition, the number of roses purchased cannot exceed $\frac{50}{3}$. Therefore the number of roses purchased must be one of the even integers between 0 and 16, inclusive. This gives 9 possibilities for the number of roses purchased, and consequently 9 possibilities for the number of bouquets.

2017A

5. **Answer (B):** Each of the 20 people who know each other shakes hands with 10 people. Each of the 10 people who know no one shakes hands with 29 people. Because each handshake involves two people, the number of handshakes is $\frac{1}{2}(20 \cdot 10 + 10 \cdot 29) = 245$.

2018B

5. **Answer (D):** The number of qualifying subsets equals the difference between the total number of subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ and the number of such subsets containing no prime numbers, which is the number of subsets of $\{4, 6, 8, 9\}$. A set with n elements has 2^n subsets, so the requested number is $2^8 - 2^4 = 256 - 16 = 240$.

OR

A subset meeting the condition must be the union of a nonempty subset of $\{2, 3, 5, 7\}$ and a subset of $\{4, 6, 8, 9\}$. There are $2^4 - 1 = 15$ of the former and $2^4 = 16$ of the latter, which gives $15 \cdot 16 = 240$ choices in all.