UNIT 1 EXERCISES 1-5

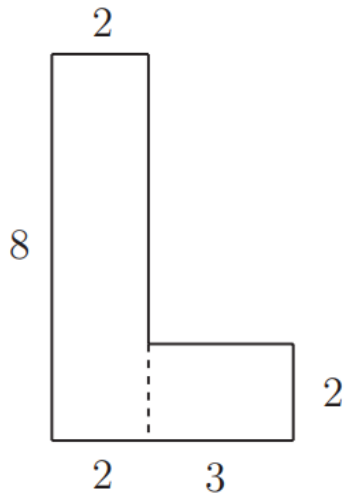
2D Geometry

2013A

1. **Answer (E):** The legs of $\triangle ABE$ have lengths $AB = 10$ and BE. Therefore $\frac{1}{2} \cdot 10 \cdot BE = 40$, so $BE = 8$.

2010B

2. **Answer (A):** The region consists of two rectangles: an 8-by-2 rectangle, and a 3-by-2 rectangle. The desired area is $8 \cdot 2 + 3 \cdot 2 = 22$.



2011A

2. **Answer (E):** The sum of her first 5 test scores is 385, yielding an average of 77. To raise her average to 80, her 6^{th} test score must be the difference between $6 \cdot 80 = 480$ and 385, which is 95.

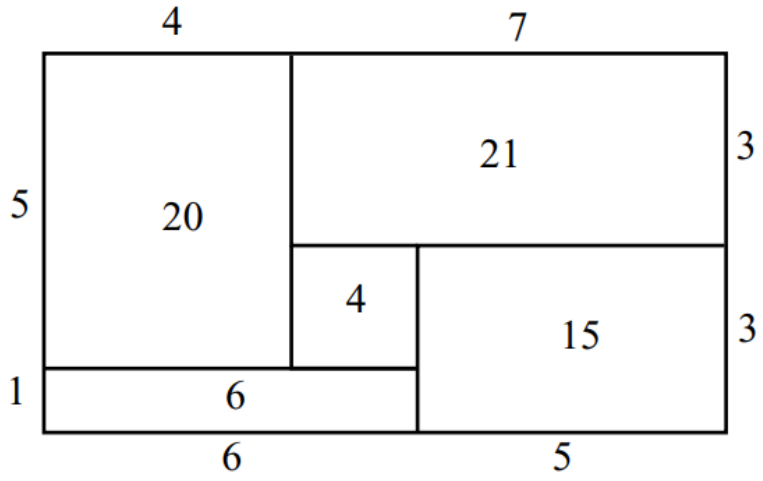
2012B

2. **Answer (E):** The width of the rectangle is the diameter of the circle, so the width is $2 \cdot 5 = 10$. The length of the rectangle is $2 \cdot 10 = 20$. Therefore the area of the rectangle is $10 \cdot 20 = 200$.

2003B

3. (A) To minimize the cost, Rose should place the most expensive flowers in the smallest region, the next most expensive in the second smallest, etc. The areas of the regions are shown in the figure, so the minimal total cost, in dollars, is

$$(3)(4) + (2.5)(6) + (2)(15) + (1.5)(20) + (1)(21) = 108.$$



2007B

3. **Answer (D):** Since $OA = OB = OC$, triangles AOB, BOC, and COA are all isosceles. Hence

$$\angle ABC = \angle ABO + \angle OBC = \frac{180^\circ - 140^\circ}{2} + \frac{180^\circ - 120^\circ}{2} = 50^\circ.$$

OR

Since

$$\angle AOC = 360^\circ - 140^\circ - 120^\circ = 100^\circ,$$

the Central Angle Theorem implies that

$$\angle ABC = \frac{1}{2}\angle AOC = 50^\circ.$$

2010A

3. **Answer (E):** Let s equal the side length of the square. Because half of the area of the rectangle is in the square, $\frac{1}{2}AB = s$. Because one fifth of the square's area is in the shaded region, $s = 5 \cdot AD$. Therefore $\frac{1}{2}AB = 5 \cdot AD$, and $\frac{AB}{AD} = 10$.

2008B

4. **Answer (D):** The measure of $\angle COD$ is $180° - 30° - 45° = 105°$. Therefore the ratio of the area of the sector to the area of the circle is $\frac{105}{360} = \frac{7}{24}$.

2009B

4. **Answer (C):** Each triangle has leg length $\frac{1}{2} \cdot (25 - 15) = 5$ meters and area $\frac{1}{2} \cdot 5^2 = \frac{25}{2}$ square meters. Thus the flower beds have a total area of 25 square meters. The entire yard has length 25 and width 5, so its area is 125. The fraction of the yard occupied by the flower beds is $\frac{25}{125} = \frac{1}{5}$.

2002A

5. **(C)** The large circle has radius 3, so its area is $\pi \cdot 3^2 = 9\pi$. The seven small circles have a total area of $7\left(\pi \cdot 1^2\right) = 7\pi$. So the shaded region has area $9\pi - 7\pi = 2\pi$.

2005B

5. **(A)** The four white quarter circles in each tile have the same area as a whole circle of radius $1/2$, that is, $\pi(1/2)^2 = \pi/4$ square feet. So the area of the shaded portion of each tile is $1 - \pi/4$ square feet. Since there are $8 \cdot 10 = 80$ tiles in the entire floor, the area of the total shaded region in square feet is

$$80\left(1 - \frac{\pi}{4}\right) = 80 - 20\pi.$$

2014B

5. **Answer (A):** Denote the height of a pane by $5x$ and the width by $2x$. Then the square window has height $2 \cdot 5x + 6$ inches and width $4 \cdot 2x + 10$ inches. Solving $2 \cdot 5x + 6 = 4 \cdot 2x + 10$ gives $x = 2$. The side length of the square window is 26 inches.